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Challenging the Efficient Market Hypothesis: Multifractal Insights into Price – Volume Cross-Correlations in the S&P 500

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Abstract*:* This study investigates the multifractal behaviour of prices, trading volume, and their cross-correlations in the S&P 500 index over the 2004–2024 period. To this end, we employ an integrated framework that combines the Bai–Perron structural break test with Multifractal Detrended Fluctuation Analysis (MFDFA) and Multifractal Detrended Cross-Correlation Analysis (MFDCCA). MFDFA is employed to detect scale-dependent long-range dependence and multifractality within individual time series, while MFDCCA extends this framework to examine multifractal cross-correlations between price and trading volume across different time scales. The structural break analysis reveals five endogenous break points, leading to six distinct market segments and allowing market dynamics to be examined on a segment-specific basis. The empirical evidence shows that both price and volume series display multifractal behaviour throughout the sample, although the intensity of multifractality varies across segments. By contrast, price–volume cross-correlations tend to exhibit broader and more asymmetric multifractal spectra, pointing to stronger nonlinear dependence and greater structural complexity in joint dynamics. Importantly, these results should not be interpreted as evidence of a permanent breakdown in market efficiency. Rather, they suggest that deviations from weak-form efficiency are time-varying and closely linked to changing market conditions, in line with the Adaptive Market Hypothesis and the Fractal Market Hypothesis. Overall, the joint analysis of price, volume, and their multifractal cross-structure within a structural-break-aware setting offers new insights into segment-dependent information transmission and the evolving nature of market efficiency in a major benchmark index.

Keywords: Bai-Perron; long memory; adaptive market hypothesis; price-volume cross-correlation; structural breaks.

JEL classification: G14; C22; C58.

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1. INTRODUCTION

The efficiency of capital markets is a subject that has been extensively discussed in the financial economics literature. Under the Efficient Market Hypothesis (EMH) developed by Fama (1965, 1970), market prices are believed to reflect all available information, thereby precluding the possibility of investors achieving systematic abnormal returns. According to the weak form of the EMH, past price information is insufficient to predict future price movements and therefore prices follow a random walk (Samuelson, 1965). This perspective has served as the foundational basis for numerous conventional financial theories.

Nevertheless, as demonstrated in the extant behavioural finance literature, which has evolved considerably since the 1980s, it has become evident that individuals do not invariably act in a manner that is consistent with rationality. Concurrently, market prices have been shown to deviate from time to time due to factors such as emotions, biases, and bounded rationality (Kahneman and Tversky, 1979; Shiller, 2003). This perspective posed a challenge to the deterministic nature of the EMH, suggesting that market behaviour should be explained by more complex structures.

In this context, the Adaptive Market Hypothesis (AMH), developed by Lo (2004), provides an important conceptual leap. According to AMH, market efficiency is not a static phenomenon; rather, it is a time-varying and evolutionary process. In essence, while a market may exhibit efficiency during certain periods, it may deviate from efficiency during other periods.

Concurrently, the non-linear and complex nature of financial time series has garnered mounting attention. In this regard, Peters (1994) and subsequent studies have posited that financial time series may manifest characteristics such as self-similarity and long memory, which may contradict the EMH. The concept of "long memory" in financial markets signifies the tendency of past price movements to exert influence on future price movements. This phenomenon can be interpreted as a manifestation of market inefficiency (Mandelbrot, 1972; Peters, 1994; Di Matteo, 2007).

In this framework, fractal geometry and multifractal analysis methods emerge as potent instruments for elucidating the intricate dynamics of financial time series. The notion of fractal structures was initially articulated by Mandelbrot (1963). It was observed that a multitude of natural phenomena (e.g., snowflakes, river deltas, tree branches) exhibit a propensity to manifest as similar forms to themselves. The observation of similar structures in financial time series (Wang *et al.*, 2009) is important for understanding complex market dynamics that cannot be explained by classical models. The fractal dimension is determined by how the object or time series fills its space (Peters, 1996). The Fractal Market Hypothesis (FMH) emphasises the impact of liquidity and investment horizon on investors' behaviour (Peters, 1994). Peters (1994, 1996) points out that the next data in a financial time series is not completely independent of the previous ones, i.e. previous data may be a carrier of information.

On the other hand, besides price series, other market indicators such as trading volume can also be a carrier of information (Karpoff, 1987; Gopikrishnan *et al.*, 2000). The relation between trading volume and security prices has been studied for over 40 years since (Osborne) suggested in 1959 that the variance of security prices could be modelled as a diffusion process whose variance depends on the number of transactions (Stošić *et al.*, 2015). The interplay between price and volume provides important clues about market microstructure and investor behaviour. Therefore, analysing not only prices but also trading volume in terms of multifractal structure allows for a more holistic assessment of market efficiency (Jiang *et al.*, 2019; Açikgöz *et al.*, 2024).

This study contributes to the literature by offering a structural-break-aware joint multifractal analysis of prices, trading volume, and their cross-correlations for the S&P 500 index over the 2004–2024 period. Existing studies have typically focused either on price dynamics alone or on price–volume relationships examined through linear or single-scale frameworks. In contrast, the present study combines Multifractal Detrended Fluctuation Analysis (MFDFA) and Multifractal Detrended Cross-Correlation Analysis (MFDCCA) within a framework that explicitly incorporates endogenously identified structural breaks using the Bai–Perron procedure. Specifically, MFDFA captures the presence of multifractality and long-memory behaviour in individual financial series, whereas MFDCCA allows the investigation of scale-dependent cross-correlations between price and volume dynamics. This approach allows market dynamics to be analysed at the segment level and provides a clearer view of how information is transmitted through prices, trading activity, and their interaction under different market conditions. The findings do not indicate a permanent breakdown of the Efficient Market Hypothesis. Instead, they reveal time-varying departures from weak-form efficiency, reflected in persistent and heterogeneous multifractal structures. These patterns are consistent with the Adaptive Market Hypothesis and the Fractal Market Hypothesis. Overall, the evidence shows that price–volume cross-correlations tend to display stronger and more heterogeneous multifractal characteristics than individual series, underscoring the role of joint dynamics in shaping market behaviour and offering new insights into the evolving nature of market efficiency. While existing multifractal studies on the S&P 500 largely focus on price dynamics in isolation or on single-scale correlations, empirical evidence on the joint multifractal structure of price and trading volume – particularly in the presence of structural breaks – remains limited.

The primary objective of this study is to examine the joint multifractal dynamics of price and trading volume in the S&P 500 index, with particular attention to how these dynamics evolve across structurally distinct market regimes.

The empirical findings reveal persistent multifractality and non-linear cross-correlations between price and volume, whose intensity and asymmetry vary across structural break segments, providing evidence inconsistent with the strict form of the Efficient Market Hypothesis.

Accordingly, the analysis is guided by the following research questions:

Q1*: Do price and trading volume series of the S&P 500 exhibit multifractal and scale-dependent dynamics across market segments identified by endogenous structural breaks?*

Q2: *Does the multifractal structure of price–volume cross-correlations differ systematically from those of individual price and volume series, and does it exhibit stronger nonlinearity and heterogeneity?*

Q3: *Do segment-specific multifractal patterns provide evidence of time-varying departures from weak-form market efficiency, in line with the Adaptive Market Hypothesis and the Fractal Market Hypothesis?*

The present study is predicated on the assumption that information flow may give rise to multifractal structures in volume dynamics, as well as its subsequent impact on market price. Consequently, the analysis relies on Multifractal Detrended Fluctuation Analysis (MFDFA) to capture scale-dependent long-range dependence in non-stationary time series, together with Multifractal Detrended Cross-Correlation Analysis (MFDCCA), which extends this framework to examine multifractal cross-correlations between pairs of non-stationary series. Using this combined approach, the multifractal properties of price and trading volume are analysed alongside the structure of their interaction. In this parallel, our research will generate information that will assist policymakers in comprehending market functionality and will benefit market participants, such as investors and portfolio managers, who aim to make forecasts.

While the methodological foundation of this study is closely related to the multifractal framework employed by Patil and Rastogi (2020), the present analysis departs from that approach in several important respects. Rather than concentrating exclusively on price-based measures of efficiency, it explicitly incorporates trading volume and examines price–volume cross-correlations within a multifractal setting. In addition, the analysis moves beyond static price dynamics by exploring how joint multifractal structures evolve across endogenously identified structural breaks. The sample period is also extended to cover more recent episodes, including the Covid-19 shock and the subsequent post-pandemic recovery, which allows for a broader and more realistic assessment of segment-dependent market efficiency. Together, these extensions offer additional insights into information transmission mechanisms and the changing complexity of market dynamics that are not fully captured in earlier studies. Although the study adopts an integrated MFDFA–MFDCCA framework combined with the Bai–Perron structural break procedure, it is worth noting that alternative multifractal methodologies could further strengthen robustness assessment. For instance, future research may draw on cross-multifractal spectrum (CMFS) approaches or complementary structural break detection techniques to validate and extend the segment-dependent price–volume interactions documented here. Such extensions would enable a deeper examination of nonlinear dependence across market segments, while leaving the core findings of the present study unchanged. Accordingly, rather than replicating an existing framework, the present study extends prior multifractal analyses by explicitly incorporating trading volume and price – volume interactions into a regime-dependent setting, thereby offering a broader perspective on market efficiency dynamics.

2. LITERATURE REVIEW

Research on the efficiency of capital markets has historically been shaped around the Efficient EMH and criticisms against this hypothesis. Fama (1965) argues that financial markets are characterized by weak efficiency, thereby implying that historical price movements are inadequate for predicting future prices. However, subsequent years have witnessed the emergence of alternative approaches that contend with the notion of market prices being entirely random. A seminal example of this phenomenon is the AMH, proposed by Lo (2004). According to this hypothesis, markets adapt to time-varying conditions; therefore, efficiency is not constant but cyclical. In this framework, studies suggesting that financial series may contain not only short-run but also long-range dependence have also become prominent in the literature. In particular, Peters (1994) advanced the notion that financial time series may exhibit fractal characteristics and investigated the "self-similarity" phenomenon by implementing the fractal geometry concept initially developed by Mandelbrot (1989) The objective of this study is to examine financial data.

A considerable body of research has been dedicated to the empirical evaluation of this theoretical framework through the utilization of numerical methodologies. For example, Wang *et al.* (2009), in their study on the Shenzhen Stock Exchange, used the MFDFA method and found that the Hurst exponent changes over time and the market becomes increasingly efficient. Similarly, Oprean-Stan *et al.* (2014), in their research examining the market structures of different countries through R/S analysis, found that the Brazilian market is closest to the random walk hypothesis, whereas the Chinese, Estonian and Romanian markets have fractal characteristics. Hiremath and Narayan (2016) also investigated the time-varying efficiency level on Indian stock markets and found that long memory varies over time and the market tends towards efficiency. In a different approach, Kumar *et al.* (2017) test the FMH using wavelet-based analysis on nine Asian foreign exchange markets and show that short-term investor behaviour increases significantly during crisis periods. Lin *et al.* (2018), analysed a total of 34 indices from different continents, and showed that market structures are geographically differentiated with detrended cross-correlation coefficient, and small and large fluctuations have independent identities. On the other hand, Moradi *et al.* (2021), in their analysis conducted with the L-Co-R algorithm in Tehran and London stock markets, found that while a fractal structure was detected in the Iranian market, this structure was not observed in the London market. Tiwari *et al.* (2019) examined a total of 10 developed and emerging markets with the MFDFA method and found that most of these markets have a multifractal structure and that efficiency increases not in the short term but rather in the long term. Ji *et al.* (2020) analysed the change in efficiency in the soy futures market during the trade tensions between the US and China with the DMCA and MFDFA methods and found that markets became more persistent but less correlated during the conflict period.

Among the recent studies Metescu (2022), applied R/S analysis on the price series of a publicly offered company in the Romanian market and supported the validity of the FMH with the Hurst exponent values obtained. Karaömer (2022), tested long memory with ARFIMA and ARFIMA-FIGARCH models in the markets of Mexico, Indonesia, Nigeria and Turkey, known as MINT countries, and argued that market returns in these countries are predictable and have a fractal structure. Açikgöz *et al.* (2024) examined the relations between green bonds and commodity markets with the MFDCCA method and found that both return and volatility-based cross-correlations between these assets are long-term and power law dependent. Moreover, persistence in small fluctuations and antipersist tendencies in large fluctuations are found to be dominant. Among recent studies, Masseran and Safari (2025) used MFDFA and MFDCCA methods to analyse the multifractal properties of exchange rates and reported that their results revealed that MYR/USD, MYR/RMB, MYR/GBP and MYR/CAD exchange rates exhibit scale-invariant properties, indicating the existence of a multifractal structure in these temporal series. According to the study, some pairs, including MYR/USD-MYR/CAD, MYR/USD-MYR/GBP, MYR/RMB-MYR/GBP, MYR/RMB-MYR/CAD and MYR/GBP-MYR/CAD, show weaker multifractal features in their cross-correlations. Finally, Kojić *et al.* (2025) conduct a comprehensive examination of the complexity, efficiency, and sectoral interdependencies associated with the S&P Global BMI indices amid significant global events, notably the Covid-19 pandemic and the Russia-Ukraine conflict. Their findings underscore substantial disparities in information efficiency across various sectors. Furthermore, the MFDCCA highlights the fragile nature of cross-correlations in relation to geopolitical risk. Notably, the consumer staples and energy sectors exhibit stable persistence, whereas the Information Technology sector demonstrates heightened sensitivity in terms of its complexity.

Despite the growing body of research on multifractality and market efficiency, several important gaps remain. Much of the empirical literature continues to focus primarily on price dynamics, with trading volume either omitted altogether or examined in isolation, even though it plays a central role in the transmission of market information. Studies that do consider price–volume relations typically rely on linear correlation measures or single-scale approaches, which are not well suited to capturing nonlinear and scale-dependent interactions. Moreover, although multifractal techniques such as MFDFA or MFDCCA are increasingly used, structural breaks are seldom incorporated explicitly, especially in analyses of major benchmark indices like the S&P 500. Although Patil and Rastogi (2020) study presents research within the framework of price, volume, price-volume cross-correlation, and structural breaks, these researchers examined the Sensex index for the Indian market, not the S&P 500. Beyond this contribution, the broader literature includes studies that connect trading activity to market efficiency and dependence structures, yet typically without combining all three elements – (i) price and volume jointly, (ii) multifractal cross-correlation, and (iii) endogenous regime segmentation via structural breaks – within a single S&P 500 setting.

By jointly examining the multifractal behaviour of prices, trading volume, and their cross-correlations within a structural-break-aware framework, this study seeks to address these limitations and to provide a segment-specific perspective on market dynamics and efficiency over a long time horizon.

3. METHODOLOGY

Various quantitative methods have been developed in the literature to analyse long memory structures and market efficiency in financial time series. Lo (2004) argued that the R/S method commonly used by Peters (1994, 1996) seems to be highly sensitive to short-term autocorrelation and non-stationarity, which may lead to biased estimation of long memory parameters. To overcome the disadvantages, Peng *et al.* (1994) introduced DFA (Detrended Fluctuation Analysis) to study the fractal structure of molecular chains of deoxyribonucleic acid (DNA). Based on the DFA method, Kantelhardt *et al.* (2002) proposed the multifractal detrended fluctuation analysis (MFDFA) for the first time to describe the multifractal features of time series under different time scales (Cao *et al.*, 2018). DFA and its multifractal generalisation MFDFA have since been widely used to detect long-range auto-correlations in financial markets, including stock markets, foreign exchange market and gold market (Ma *et al.*, 2013).

Various techniques have been developed to analyse fractal and multifractal properties in time series. According to Stošić *et al.* (2015), MFDFA and MFDCCA have shown to be powerful tools for analysing the multifractal behaviour of non-stationary time series.

In this study, it is aimed to detect structural breaks with the procedure developed by Bai and Perron (2003) and to apply MFDFA and MFDCCA to determine the long memory in each time series to be obtained with these results.

3.1. Bai and Perron structural break test

In analysing financial markets, the accurate identification of structural breaks is critical for understanding market dynamics and examining changes in market efficiency over time.

This study builds on the methods developed by Bai and Perron (2003) to identify and analyse structural breaks. Then, using the sequential test procedure and the method of estimating break dates proposed by Bai and Perron (2003), potential break points in market data will be identified. The identification of structural breaks allows us to understand how market efficiency changes over time and reveals how market efficiency differs before and after structural breaks. It also analyses the relation between structural breaks and AMH and shows how markets adapt to changes in market conditions.

The Bai and Perron (2003) procedure tests for fluctuation due to stability in the regression modelling given in the figure below:

$$y_i = X_i^T \beta + u_i \qquad (1)$$

The procedure starts with the assumption that there are *m* breaks, which equals *m+1* sub-sections to identify breaks, and y_i is the dependent variable observed at time *i*.

$$y_i = X_i^T \beta + u_i, \qquad (i = i_{(j-1)} + 1, \dots, i_j, \qquad j = 1, \dots, m+1) \qquad (2)$$

The coefficients are transitioned from one balanced regression to another, as shown in Equation (2). The process selects sub-sections that yield the minimum Residual Sum of Squares (RSS) and the lowest Bayes Information Criterion (BIC) value. j is the number of sub-sections, β is the vector of coefficients (Bai and Perron, 2003; Patil and Rastogi, 2020). Following standard practice in the structural break literature, the breakpoint test was implemented using a constant-only specification. The trimming parameter was set to 0.15 in order to ensure a sufficient number of observations within each segment, while the maximum number of structural breaks was restricted to five and determined endogenously based on information criteria. This configuration ensures statistically reliable segmentation. More specifically, the Bai–Perron procedure is applied within a mean-shift (constant-only) regression framework, where the dependent variable is modelled as a piecewise constant process subject to multiple structural breaks. This specification focuses on detecting changes in the unconditional mean of the series, which is particularly suitable for identifying regime shifts in financial time series. Following Bai and Perron (2003), a trimming parameter of 0.15 is adopted to ensure that each segment contains a sufficient number of observations for reliable estimation. The maximum number of structural breaks is determined endogenously based on information criteria, with the selected specification identifying up to five breakpoints. Break dates are selected using a sequential testing procedure based on supF-type statistics, allowing for a statistically consistent identification of multiple structural breaks.

3.1.1. MFDFA and MFDCCA

Prior to the implementation of the MFDFA and MFDCCA procedures, the financial time series are transformed following standard practice in the multifractal literature. Let P_t denote the closing price of the S&P 500 index at time *t*. The logarithmic price series is defined as $X_t = \ln(P_t)$, which serves as the primary input for the multifractal analysis. Trading volume is denoted by V_t, and its logarithmic transformation, which is used in the subsequent analysis, is defined as $Y_t = \ln(V_t)$ to mitigate scale effects and heteroskedasticity. Both transformed series are demeaned prior to analysis to ensure compatibility with the mean-shift framework adopted in the structural break procedure. In the MFDFA framework, the integrated profile is constructed from each transformed series and analysed separately to capture scale-dependent long-range dependence. In the MFDCCA framework, the joint multifractal behaviour is examined by applying the procedure to the paired series (X_t, Y_t) allowing the investigation of scale-dependent cross-correlations between price and trading volume.

In the analysis of financial markets, the effects of market efficiency and structural breaks can be examined by using multiple fractal analysis methods as frequently encountered in the literature. In this study, MFDFA and MFDCCA methods are used to examine the multidimensional structure of market efficiency and cross-correlations between different financial series in detail (Kantelhardt *et al.*, 2002; Podobnik and Stanley, 2008; Patil and Rastogi, 2020). Detrended Fluctuation Analysis (DFA) was introduced by Kantelhardt *et al.* (2002) to effectively address the challenges associated with time series data that exhibit trends. The methodology of MFDFA further refines this approach by calculating the detrended residuals of a time series that resembles a random walk. This technique involves partitioning the series into segments of equal size, ensuring that these segments remain non-overlapping. Building upon this foundation, MFDCCA was developed by Zhou (2008), drawing insights from the earlier work of Podobnik and Stanley (2008), thereby extending the capabilities of MFDFA into the realm of multivariate time series analysis. Jiang *et al.* (2019) then described this method (Patil and Rastogi, 2020). In this study, price and volume series will be analysed with MFDFA and cross-correlation of price and volume will be examined with MFDCCA.

3.1.2. MFDFA

The MFDFA method is designed to unveil the multifractal nature of time series by examining the degree of fluctuations occurring at varying time scales. In this method, scale-dependent fluctuation functions are calculated by removing the trends in the series and the Hurst exponent and multifractal spectrum are obtained from these functions.

The MFDFA methodology involves the computation of trended residuals derived from a random walk-like time series, which is systematically partitioned into non-overlapping segments of uniform length. Within each segment, the root mean square of the residuals is determined, and the resultant mean is designated as the trend-free fluctuation function for that particular subsegment. This approach facilitates the examination of the intrinsic fluctuation characteristics of the time series under study, thereby enhancing the understanding of its multifractal nature. In addition, the qth order of the trend-free fluctuation function of this subsegment sub-section is calculated. The relation between the segment size and the fluctuation function is subject to a power law. This power law relation is captured in the Hurst exponent and described by a very fractal spectrum. The above steps are performed using the following equations:

1. The detrended residuals are computed using the following equation:

$$\varepsilon(i) = \mathcal{X}_t - (\widehat{\mathcal{X}}) \tag{3}$$

2. The fluctuation function is computed as the RMS (Root-Mean-Square) of the trend-reduced residuals:

$$[F_v(s)]^2 = \frac{1}{s}\sum_{j=1}^{s}\left[\varepsilon\big((v-1)s\big)+j\right]^2 \tag{4}$$

3. The q. order of the fluctuation function is computed for a a specified value of q.

$$[F_v(s)]^q = \frac{1}{s}\sum_{j=1}^{s}\left[\varepsilon\big((v-1)s\big)+j\right]^q \tag{5}$$

4. The Law of Power relation is defined;

$$F_q(s) \sim s^{h(q)} \tag{6}$$

The singularity spectrum $(f(\alpha))$ characterizes the fractal dimension of these multifractals, where α is represents the singularity strength. This fractal dimension is formulated in terms of $f(\alpha)$ can be computed using the following equation:

$$f(\alpha) = q\alpha - \tau(q) \tag{7}$$

where $\tau(q)$ is expressed in terms of the generalised Hurst exponent (*H(q)*). *H(q)* is calculated;

$$H(q) = \lim_{q' \to q}\left[\frac{\tau(q')+1}{q'}\right] \tag{8}$$

or;

$$\tau(q) = q^{*}h(q) - 1 \tag{9}$$

as above (Patil and Rastogi, 2020).

3.1.3. MFDCCA

MFDCCA, a bivariate extension of MFDFA, is employed to unveil multifractal correlations between two distinct time series. This method is particularly effective in analysing complex relations between market prices and trading volumes.

In the application of the MFDCCA method for analysing price-volume cross-correlation, a series of procedural steps analogous to those employed in the MFDFA framework are undertaken. The fluctuation function is computed utilizing the following equation:

$$F_v^2(q,s) = \frac{1}{s}\sum_{k=1}^{s}\left[X_v(k) - \widehat{X_v(k)}\right]\left[Y_v(k) - \widehat{Y_v(k)}\right] \tag{10}$$

The q. cross-correlation between the series is calculated as follows;

$$F_{xy}^2(q,s) = \left[\frac{1}{m}\sum_{v=1}^{m}F_v^q(s)\right]^{\frac{1}{q}} \tag{11}$$

and scaling relation,

$$F_{xy}(q,s) \sim H_{xy}(q) \tag{12}$$

is expressed as in equation (12) (Jiang *et al.*, 2019; Patil and Rastogi, 2020).

In this context, the study analyses both the effects of structural breaks on market efficiency and the multiple fractal correlations between price and trading volume in detail by applying MFDFA and MFDCCA methods in each segment after the structural breaks determined by the Bai and Perron (2003) procedure.

4. DATA

The financial time series analysed in this study consists of daily closing prices and daily trading volume data of the S&P 500 index. The data set covers the period between 5 January 2004 and 31 December 2024, and contains 5282 daily observations in total. This wide time span makes it possible to observe the dynamic behaviour of markets in different segments, especially crisis periods, recovery processes and stationary period within the same data set.
Data are obtained from Bloomberg Data Terminal. Raw price and volume data have been transformed by taking their natural logarithms in order to facilitate the series to meet the stationarity assumption and to obtain more reliable results in the analyses (All variables are log-transformed with natural logarithm, i.e. ln, e-based logarithm). At this point, the formula $r_t = [ln(P_t) - ln(P_{t-1})]$ for price and $V_t = [ln(V_t) - ln(V_{t-1})]$ for volume were used. Unnormalized log returns (PriceLNC) and volume changes (VolumeLNC) are employed in the MFDFA analyses for the entire dataset. This approach provides a more accurate analysis by preserving the natural structure of multifractal features and is widely used in the literature (Kantelhardt *et al.*, 2002; Carbone *et al.*, 2004; Oświęcimka *et al.*, 2006). The sufficient length of the data set made this choice possible. On the other hand, in cross-correlation-based MFDCCA analyses, normalized series are used in order to detect the common structural patterns of variables of different magnitudes more reliably. Furthermore, in all analyses (MFDFA and MFDCCA) for each segment obtained as a result of structural break tests, normalized price and volume series are employed to ensure cross-scale comparability and to reduce bias in moment calculations. Thus, methodological integrity is maintained in all segment-based analyses.

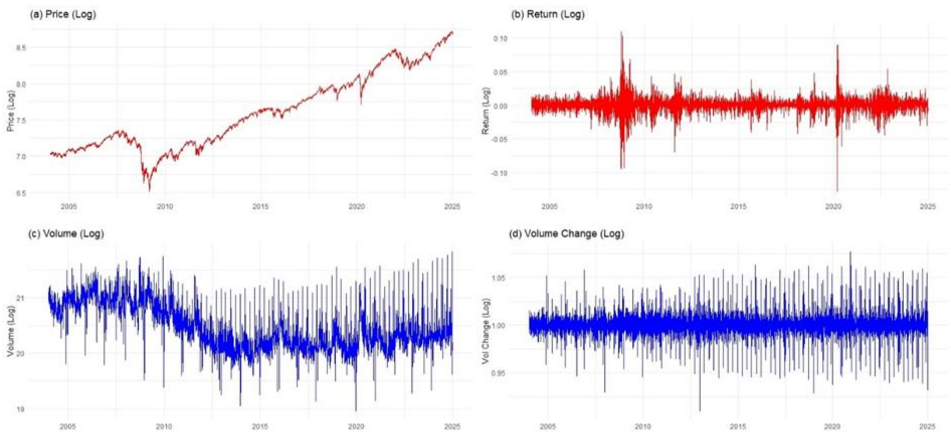


Figure no. 1 – Visualisation of the data

Figure no. 1 visualises the behaviour of the key variables in the data set over time after log transformation. The graphs provide important clues about the basic structure of the series.

In Panel (a), the logarithmic price series (Price (Log)) exhibits a long-term trend. During the 2008 global financial crisis, sharp declines are observed, followed by a recovery and a continuous upward trend. The 2008 mortgage crisis is one of the most significant structural breaks in the series. On the other hand, the impact of the Covid-19 pandemic that emerged in early 2020 was limited and the index recovered in a short time.

The logarithmic return series (Return (Log)) in Panel (b) exhibits a high-frequency volatility, with a significant increase in volatility during the crisis periods (2008 and 2020). This reveals the impact of financial stress on price volatility.

An analysis of the log volume series (Volume (Log)) in Panel (c) reveals a decreasing trend over time, followed by a flat trend and high volatility. The fluctuations in the volume data become more pronounced especially during crisis periods, and periodic increases and decreases stand out in the structure of the series.

In Panel (d), the logarithmic volume change series (Volume Change (Log)) exhibits a more stable but still volatile structure. Although shorter-term fluctuations are observed compared to prices, no clear trend is observed. This feature suggests that the series is characterized by weak continuity and more homogenous variance.

Table no. 1 - Summary statistics of the data

Variable	N	Min	Max	Median	Mean	SE_Mean	95% CI (Lower)	95% CI (Upper)	SD
PriceLN	5282	6.51698	8.71445	7.55227	7.59959	0.00713	7.58561	7.61358	0.51845
VolumeLN	5282	18.9560	21.8333	20.3422	20.4510	0.00574	20.4397	20.4622	0.41718
PriceLNC	5282	-0.12765	0.10957	0.00071	0.00032	0.00016	-0.00001	0.00064	0.01191
VolumeLNC	5282	0.90976	1.07757	0.99975	1.00007	0.00017	0.99974	1.00041	0.01257
PriceLNC_NORM	5282	-10.7428	9.1720	0.03283	0.00000	0.01376	-0.02697	0.02697	1.00000
VolumeLNC_NORM	5282	-7.18325	6.1635	0.02579	0.00000	0.01376	-0.02697	0.02697	1.00000

Note: SE and CI stand for the Standard Error and Confidence Interval of the mean

Table no. 1 presents summary statistics of the main series used in the analysis. The variables are grouped into three main categories: logarithmic level series (PriceLN, VolumeLN), logarithmic difference series (PriceLNC, VolumeLNC) and normalised logarithmic difference series (PriceLNC_NORM, VolumeLNC_NORM). This classification is important to accurately reflect the effects of the applied transformations on the analysis approach.

PriceLN and VolumeLN are obtained by taking the natural logarithm of the raw price and volume data. This transformation stabilises the variance and makes the distribution more symmetric, provided the data are positive. The high standard deviation values observed (~0.52 and ~0.42, respectively) indicate that price and volume levels vary over a wide range over time.

PriceLNC and VolumeLNC are logarithmically differenced series and represent the log return and log volume change, respectively.

The mean of PriceLNC is close to zero (0.00032), indicating that the return series does not have a statistically significant positive central tendency. The mean of VolumeLNC is very close to 1 (~1.00007), indicating a systematic central tendency due to the transformation. These difference series are the main input for direct MFDFA analyses.

PriceLNC_NORM and VolumeLNC_NORM are the z-score transformed (normalized) versions of the above difference series. This transformation is used especially in MFDCCA analyses to ensure comparability between series at different scales. This transformation is calculated as the ratio of the deviation of each observation from the mean of its series to its standard deviation and is obtained using the following formula:

$$Z_i = \frac{x_i - \bar{x}}{s} \quad (13)$$

where x_i is the observation of the series i. s is the standard deviation of the series. This type of normalisation has been used to improve inter-scale comparability between series, reduce bias in moment calculations and facilitate the accurate detection of structural patterns between variables, especially in MFDCCA. Z-score normalization has been widely applied, especially in the context of financial time series analyses. (Oświęcimka *et al.*, 2006; Podobnik and Stanley, 2008; Zhou, 2008).

The means of these normalised series are structured as zero and standard deviations as 1. The 95% confidence intervals are at the level of ±0.02697 and these values show that there is no outlier problem in the distribution of the series.

These descriptive statistics indicate that the data set is suitable for non-linear time series analyses. Especially in MFDFA and MFDCCA analyses, knowing the statistical distribution properties of the data is critical to assess the reliability of the multifractal measurements obtained. In addition, preventing variance bias in segment-based analyses with normalised series was considered as a methodological requirement for the consistency of moment-based measurements.

The fact that the data set belongs to the S&P 500 index enables the findings of this study to have implications for both the financial markets of advanced economies and global market behaviour. Since the S&P 500 reflects the performance of the 500 largest publicly traded companies in the United States, it is considered an important indicator of global investor sentiment and behaviour. Moreover, as it is a frequently analysed indicator in the literature, this study has the potential to contribute to the comparative studies conducted in different countries and in different periods.

Before analysing each segment, MFDFA and MFDCCA methods were applied for the whole series. Then, the data were sub-segmented according to the structural break points obtained by the Bai and Perron (2003) procedure and MFDFA and MFDCCA analyses were performed on each segment. This methodological distinction enables more accurate modelling of the time-varying market structure and more meaningful results within the framework of the Adaptive Market Hypothesis (Lo, 2004).

In conclusion, the data set used is considered to be structurally sound and suitable for statistical analyses, allowing to examine the multifractal nature of both long-term and cyclical market behaviour.

5. EMPIRICAL RESULTS

This section presents the empirical results obtained from the MFDFA and MFDCCA analyses for the full sample as well as for the structural segments identified by the Bai–Perron break test. The discussion does not aim to reproduce detailed graphical patterns; instead, it concentrates on the main multifractal features that distinguish prices, trading volume, and their cross-correlations across segments. Segment-specific figures and summary statistics are reported for reference, while shared patterns and segment-dependent differences are brought together in the Cross-Segment Synthesis subsection.

5.1. Structural fracture analysis

As previously stated, the methodology established by Bai and Perron (2003) was employed to identify structural breaks in the data set under consideration. The "strucchange" package in the R Studio program was utilized for the calculation. The optimal number of structural breaks was determined using the BIC criterion, and the analysis identified five break points, resulting in six segments, as illustrated in Figure no. 2.

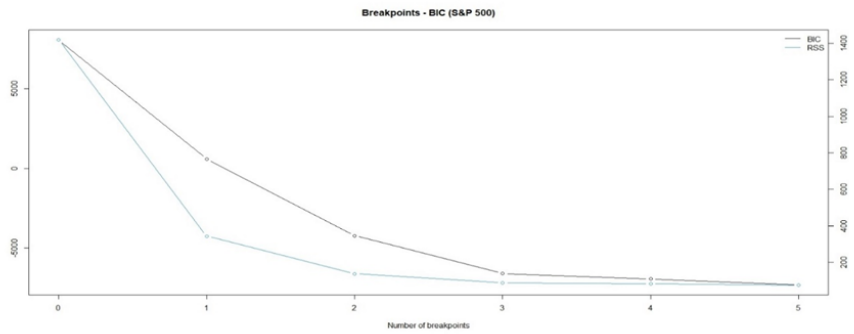


Figure no. 2 – BIC and residual sum of squares

According to the structural break analysis applied with the Bai and Perron (2003) procedure, the optimal number of breaks is determined by taking into account the Bayes Information Criterion (BIC) and Residual Sum of Squares (RSS) values. The structural break dates obtained were determined as follows: November 12, 2007 (before the 2008 Global Financial Crisis), January 5, 2011 (European Debt Crisis), March 3, 2014 (Ukraine-Russia tensions and the US Federal Reserve's tapering of asset purchases), July 11, 2017 (Trump-era global policy uncertainties), and November 12, 2020 (post-Covid-19 recovery). The sample under analysis was divided into six distinct sub-periods, and multifractal analyses were conducted on each segment.

Table no. 2 – Descriptive statistics for segments

Variable	N	Min	Max	Median	Mean	SE Mean	95% CI Mean	SD
2004-01-05-2007-11-12	971	6.9691	7.3557	7.1274	7.1455	0.0033	0.0065	0.1027
2007-11-12-2011-01-05	792	6.5170	7.3238	7.0128	7.0051	0.0061	0.0120	0.1716
2011-01-05-2014-03-03	792	7.0024	7.5280	7.2440	7.2718	0.0046	0.0089	0.1282
2014-03-03-2017-07-11	845	7.5042	7.8053	7.6394	7.6429	0.0024	0.0047	0.0700
2017-07-11-2020-11-12	843	7.7131	8.1834	7.9496	7.9586	0.0032	0.0063	0.0927
2020-11-12-2024-12-31	1038	8.1768	8.7144	8.3823	8.4018	0.0041	0.0080	0.1314

Note: Mean and standard error values indicate *Standard Error (SE)* and *Confidence Interval (CI)*, respectively. The CI value was calculated at 95% confidence interval.

Basic descriptive statistics of the log-transformed price series (PriceLN) for each structural break period are presented in the Table no. 2. The upward trend in the mean values over time reflects the long-term upward trend of the index. The higher standard deviation and lower mean values observed especially in the 2007-2011 period reveal the effects of the 2008 global financial crisis. These segment-based statistics allow for a comparative assessment of the structural characteristics of each period prior to multifractal analysis.

Figure no. 3 shows the segments obtained with the detected structural break dates.

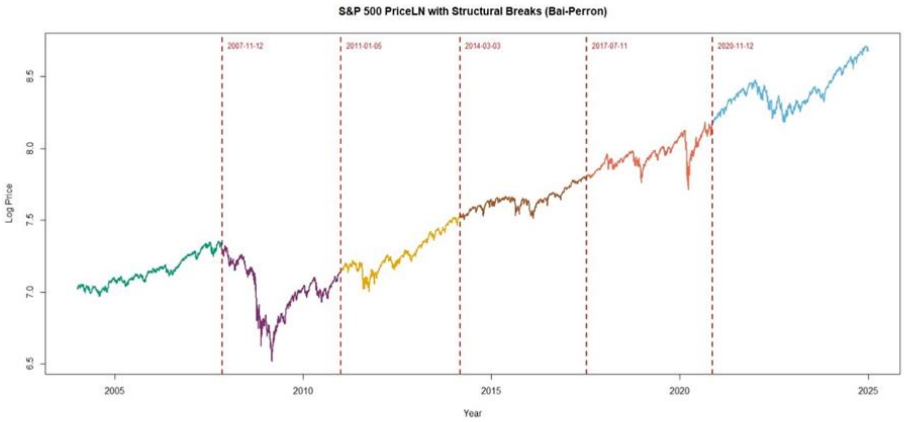


Figure no. 3 - Segments visualisation

5.2. Multifractal analysis

In this study, multifractal analyses were performed using the MFDFA package defined in the R programming language (Laib *et al.*, 2019) and the MFDCCA function created specifically for the study. In all segment-based analyses, the series were first de-averaged and normalized with the scale() function. Then, in accordance with the classical multifractal analysis approach, the cumulative sums of these normalised series were taken and integrated. This transformation enables the structural features to be revealed more accurately, especially in high-frequency and noisy series such as log-price and log-volume change. In both MFDFA and MFDCCA analyses, normalised PriceLNC and VolumeLNC series were used, and this choice allowed consistent and comparable analysis results to be obtained at multiple scales, regardless of whether the series is whole or segment-based. Multifractal analyses were performed separately for each segment obtained as a result of structural break tests. In each analysis, the segment length *s* (scale) value, which determines the scale size, was limited to 100 different scales distributed logarithmically in the range from 8 to N/2. The approach of logarithmically distributing the segment length is a common practice (Kantelhardt *et al.*, 2002; Ihlen, 2012; Al-Yahyaee *et al.*, 2018). These scales are designated as segments that do not overlap each other. The moment degree *q* is chosen in the range of -5 to +5, (Zhou, 2009; Ihlen, 2012; Takaishi, 2022), while the polynomial degree *m* used for detrending is classically 1 (linear detrending) (Suárez-García and Gómez-Ullate, 2014; Patil and Rastogi, 2020). While calculating the fluctuation functions in all MFDFA and MFDCCA analyses, the second-moment approach commonly used in the literature was adopted. In this approach, intra-segment residual variances or cross-covariances were squared and *Fq(s)* functions were calculated over generalised moments with *q-parameter*. On the other hand, absolute value-based fluctuation calculations, which are frequently encountered in MFDCCA analysis and used in some alternative applications to overcome the problem of negative *F(q)* values, are not preferred in this study. Stošić and Stošić (2025) made a comprehensive study on this subject. The second-moment (q = 2) formulation adopted in this study follows standard practice in multifractal analysis and is widely used to ensure stable estimation when higher-order moments become numerically unreliable (Kantelhardt *et al.*, 2002; Zhou, 2008; Jiang and Zhou, 2011). The squaring approximation provides both theoretical continuity and a more stable calculation of the functions $\tau(q)$, $h(q)$, α and $f(\alpha)$.

For each segment, multiple plots consisting of four panels are presented:

Panel (a): MFDFA analyses for the price series (PriceLNC)

Panel (b): MFDFA analyses for the volume series (VolumeLNC)

Panel (c): MFDCCA outputs analysing the price-volume cross relation

Panel (d): Comparative presentation of multifractal spectra of three structures

Panels (a) and (b) contain four subplots:

Fluctuation Function: Visualises the fluctuation intensity of the time series at different values of q and s. Negative q is sensitive to small fluctuations, positive q to large fluctuations.

Hurst Exponent h(q): Indicates the persistence of the series. $h(q) < 0.5 \rightarrow$ anti-persistence; $h(q) = 0.5 \rightarrow$ randomness; $h(q) > 0.5 \rightarrow$ persistence (long memory).

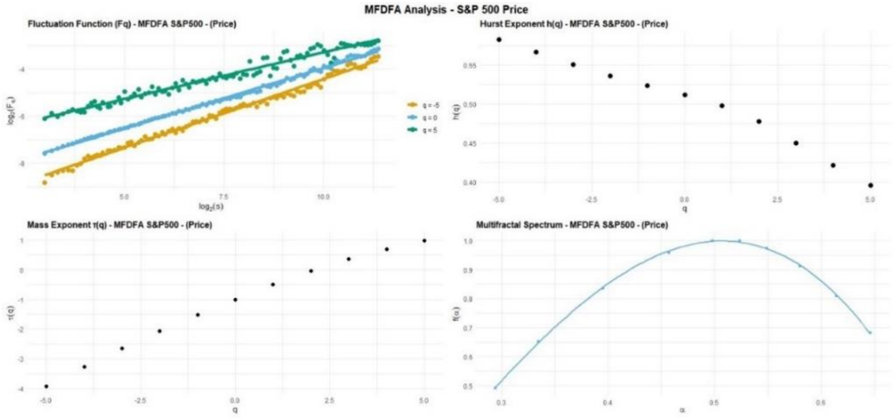
Mass (Renyi) Exponent τ(q): Multifractality is present if the plot of $\tau(q)$ against q is not linear.

Multifractal Spectrum f(α): Represents the intensity of local variations and the diversity of distributions in the series. The width of the spectrum indicates the degree of multifractal effects and the range α indicates the heterogeneity of the series.

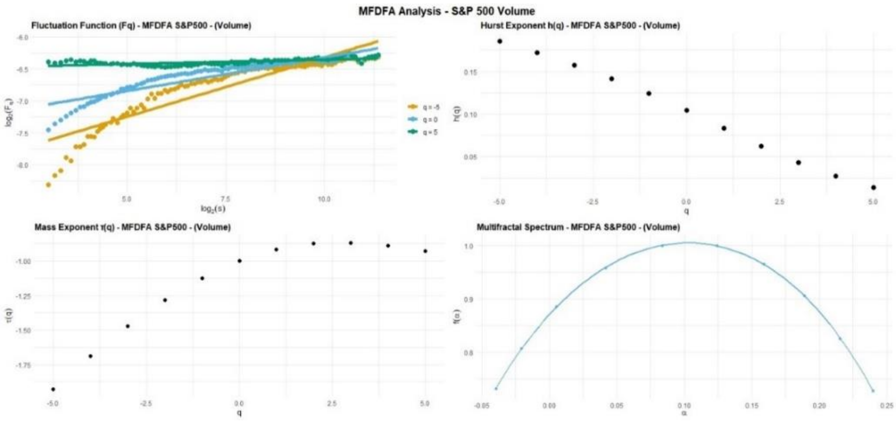
A shift of the multifractal spectrum to the left is commonly interpreted as an indication of anti-persistent behaviour, whereas a shift to the right reflects persistent dynamics (Kantelhardt *et al.*, 2002; Zhou, 2008). Moreover, the spread of $h(q)$ and α values reveals the heterogeneity and structural complexity of the series.

5.3. Whole data set

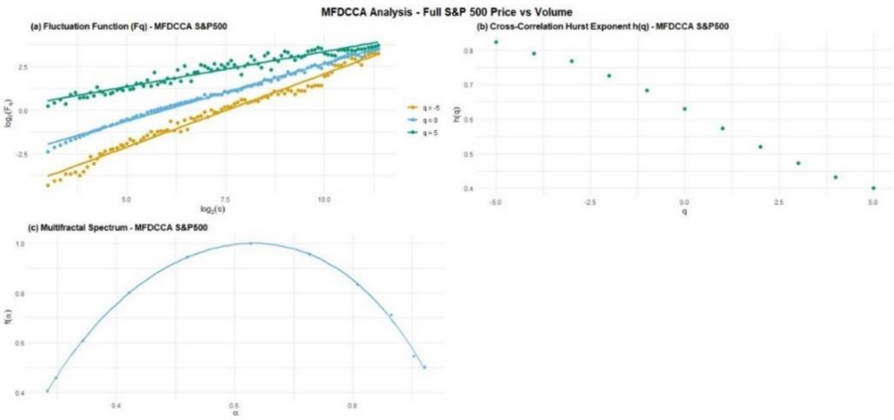
The outputs produced as a result of the analyses run for the entire data set and the graphs obtained as a result of their visualisation as described in the previous section are as follows.



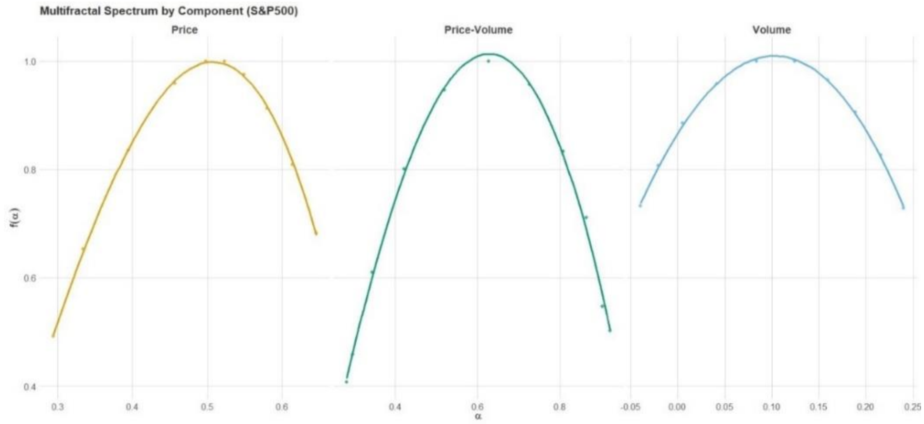
(a) - Price Spectrum



(b) - Volume Spectrum



(c) - Price- Volume Cross-Correlation Spectrum



(d) - Spectrum Comparison

Figure no. 4 – Whole data-price, volume and price-volume cross-correlation

The multifractal spectrum of the price series shown in Figure no. 4a reveals that for positive *q* values representing large fluctuations, the Hurst exponent remains below 0.5 and therefore the series exhibits anti-persist properties. This is also supported by the fact that the slopes in the fluctuation function graph are less steep for $q=5$ and steeper for $q=-5$. These results suggest that the price series tends to exhibit an anti-persistent behaviour for large fluctuations. For small fluctuations, the fact that the value of $h(q)$ is quite close to 0.5 implies that the price series can also take on a random structure from time to time.

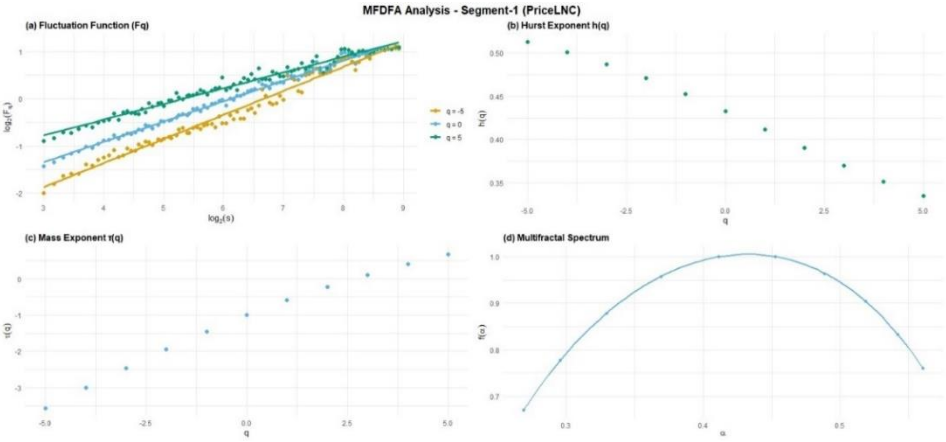
Figure no. 4b shows the results for the volume series, where it is observed that $h(q)$ values are generally below 0.2 for both large and small fluctuations. This indicates that the volume series is markedly anti-persist in both low and high scale fluctuations. Furthermore, the multifractal spectrum has a narrower and left-shifted form, implying that the volume series shows lower multifractal density.

The results of the cross-correlation analysis between price and volume in Figure no. 4c show that the interactions of both series have an anti-persist character. The $h(q)$ curve remains below 0.5 for all *q* values, indicating that there are no traces of long-range memory in the fluctuations in the cross structure and that inverse relations are prominent. The width of the spectrum curve indicates that the interactions between price and volume span a wider range of singularities and exhibit a more pronounced multifractal behaviour than the volume series.

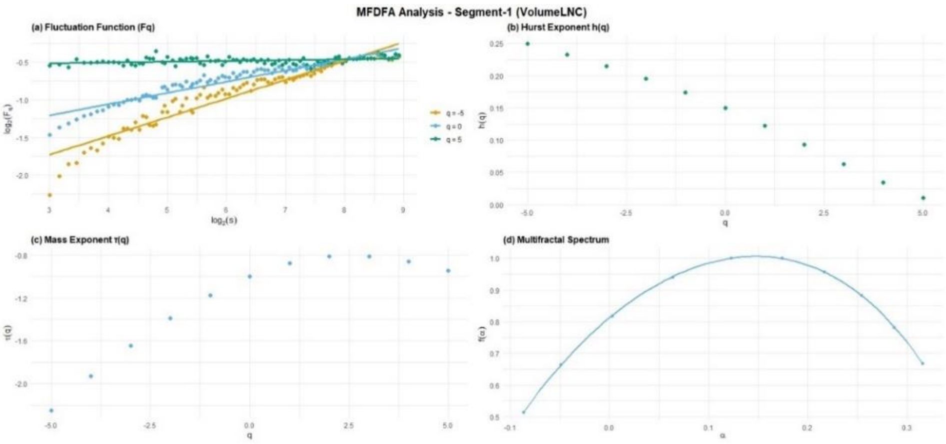
Finally, the comparative multifractal spectrum panel in Figure no. 4d shows that the price–volume cross-correlation exhibits a broader spectrum width than both the individual price and volume series. This indicates that the interaction between prices and trading activity is characterised by higher multifractal intensity and greater fluctuation diversity than either series in isolation. Moreover, the left-skewed asymmetry of the spectra suggests that anti-persistent behaviour dominates across all analysed series, consistent with the interpretation of multifractal scaling properties outlined by Kantelhardt *et al.* (2002) and Zhou (2008). Taken together, these findings indicate that the long-memory structure of price and volume dynamics in the S&P 500 index evolves over time and exhibits scale-dependent departures from the strict assumptions of weak-form market efficiency, particularly during periods of elevated volatility.

5.4. Segment 1 - segment 6

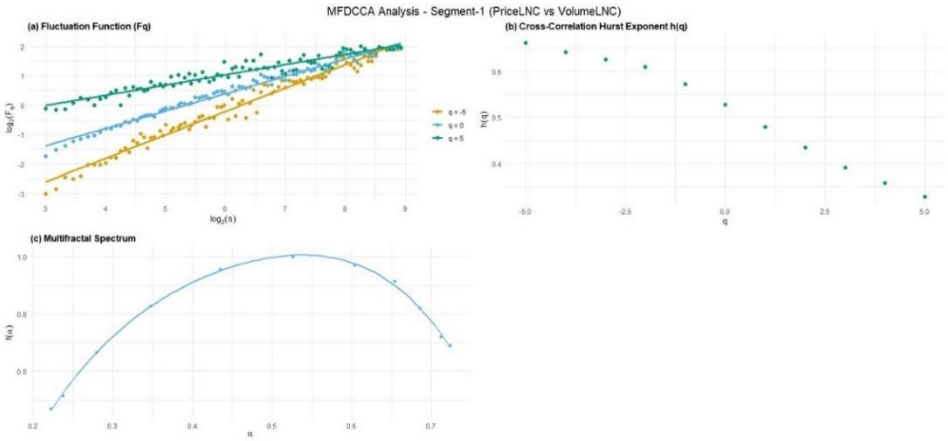
This subsection examines the multifractal characteristics of prices, trading volume, and their cross-correlations across the structural segments identified by the Bai–Perron procedure. Rather than dwelling on detailed graphical patterns, the discussion highlights the key features that distinguish each segment. A broader comparison across segments is then developed in the following subsection.



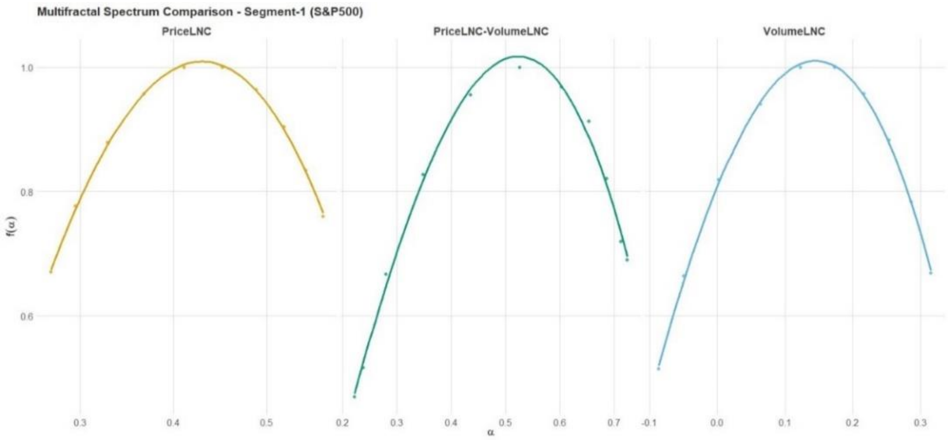
(a) - Price Spectrum



(b) - Volume Spectrum



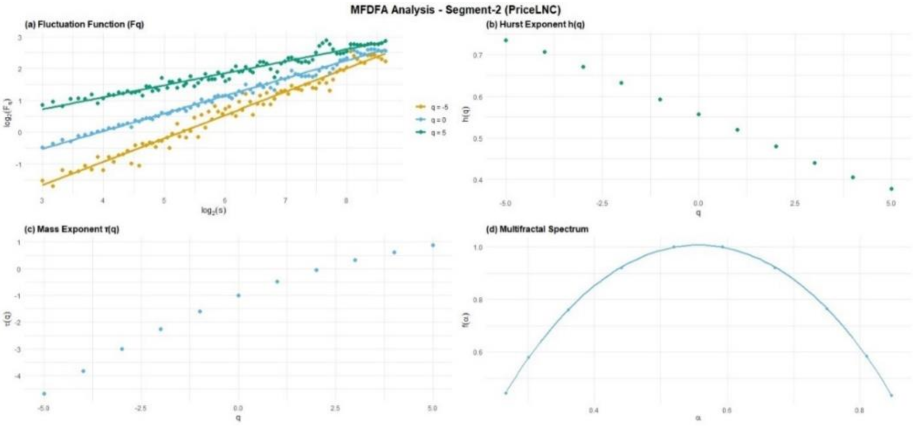
(c) - Price- Volume Cross-Correlation Spectrum



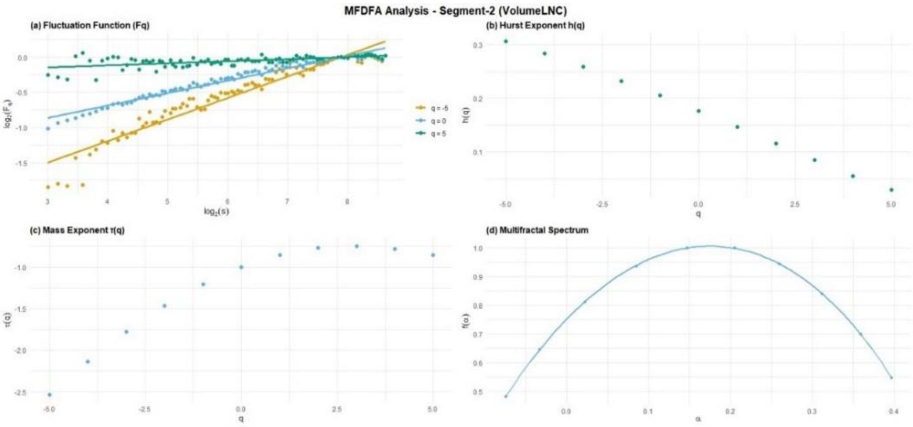
(d) - Spectrum Comparison

Figure no. 5 – Segment 1

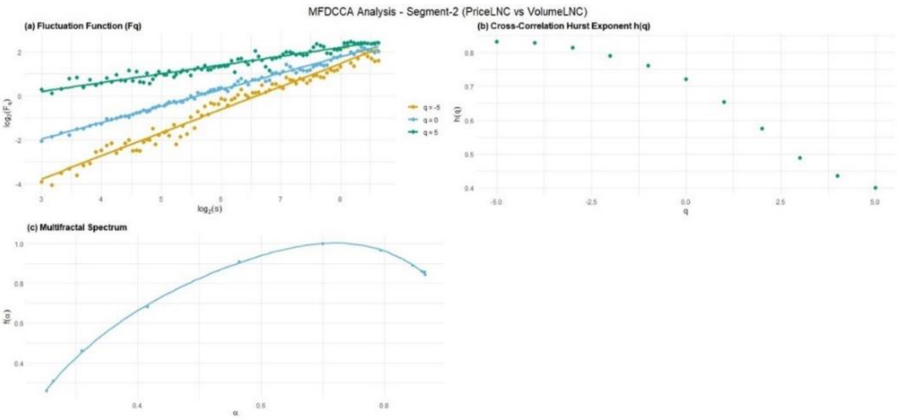
Segment 1 represents a period prior to the global financial crisis, characterised by relatively stable market conditions. During this period, price dynamics exhibited a moderate multifractal structure, and scale-dependent behaviour indicated differing responses between small and large fluctuations. On the other hand, trading volume largely exhibits anti-persistent structure characteristics, indicating that transactions are short-lived and rapidly reversible. Nevertheless, as can be seen in Figure no. 5 and Table no. 3, the price-volume cross-correlation reveals a broader and more asymmetric multifractal structure than price alone or volume alone. This suggests that even during relatively calm financial market conditions, information transmission and market adjustment processes appear to be more strongly reflected in nonlinear joint price–volume dynamics than in isolated market variables. In terms of market efficiency, these findings suggest that deviations from weak-form efficiency, while limited in magnitude during stable periods, are more readily observable in the joint price–volume domain than in individual series.



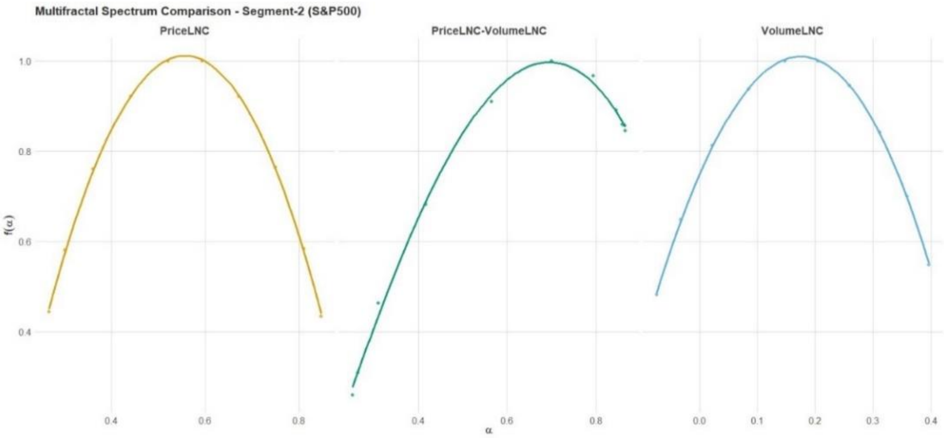
(a) - Price Spectrum



(b) - Volume Spectrum



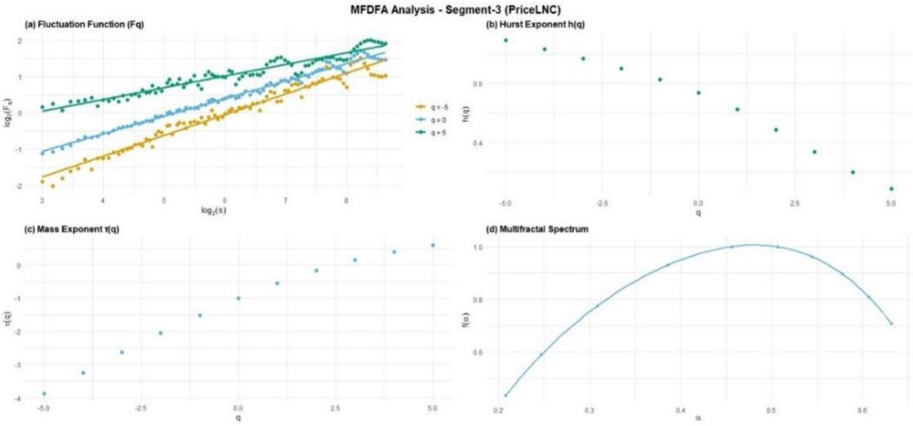
(c) - Price- Volume Cross-Correlation Spectrum



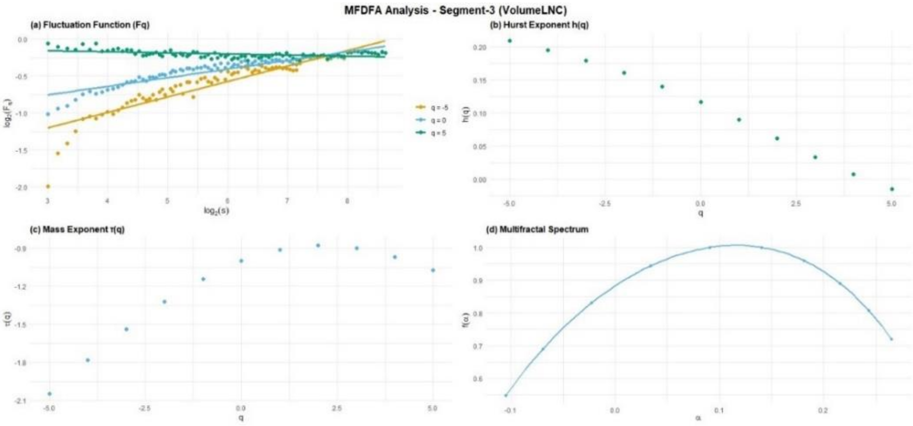
(d) - Spectrum Comparison

Figure no. 6 – Segment 2

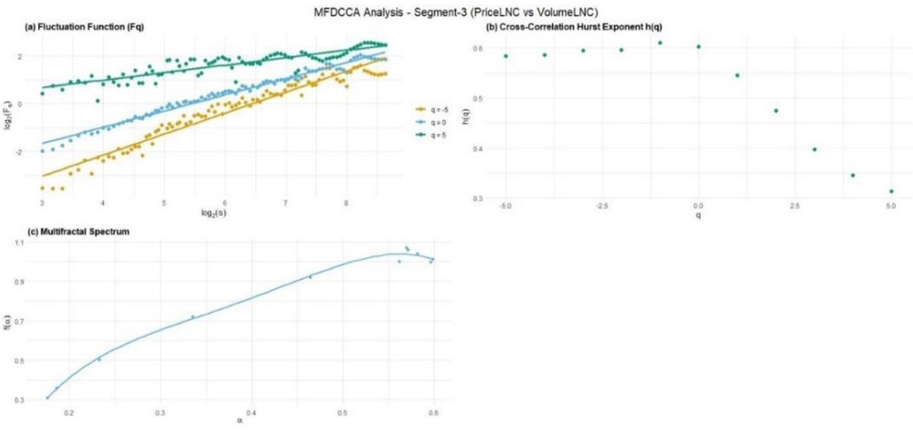
Segment 2 covers a period encompassing the outbreak of the US-centred Mortgage Crisis, during which multifractal heterogeneity intensified in market dynamics. Markets tend to exhibit unstable reactions under increasing stress. This period reflects these reactions as a segment where price movements exhibit scale dependency and behavioural differences are observed between small and large fluctuations. Trading volume continues to be predominantly anti-persistent and points to trading activity that is prone to short-term and rapid reversals despite high uncertainty. The most striking feature is the behaviour of price-volume cross-correlations. As shown in Figure no. 6 and summarised in Table no. 3, these correlations exhibit a much broader and more asymmetric multifractal structure than either prices or volumes alone. Compared to the pre-crisis period, the strengthening of this shared multifractality indicates that crisis conditions have amplified the non-linear information transfer between prices and trading activity, and that cross-dynamics are more informative than isolated series when assessing systemic market stress. In AMH terms, this segment reflects a regime where market efficiency is temporarily weakened rather than permanently impaired, and adaptive behaviours become more visible through price-volume interactions.



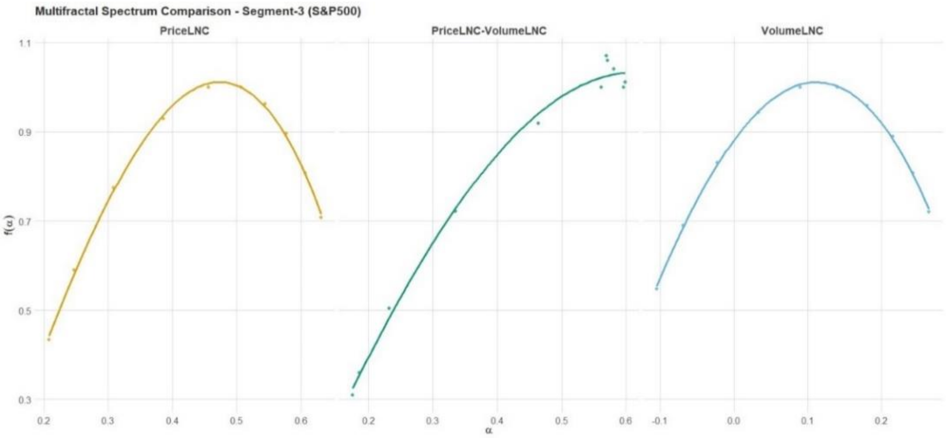
(a) - Price Spectrum



(b) - Volume Spectrum



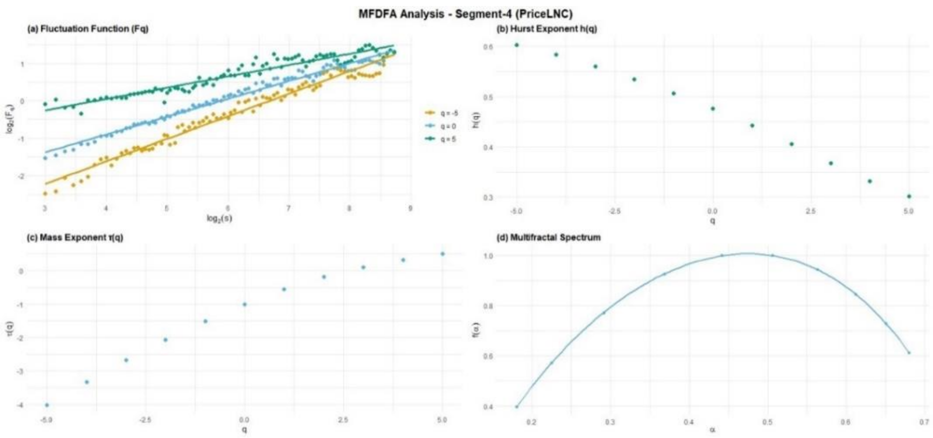
(c) - Price- Volume Cross-Correlation Spectrum



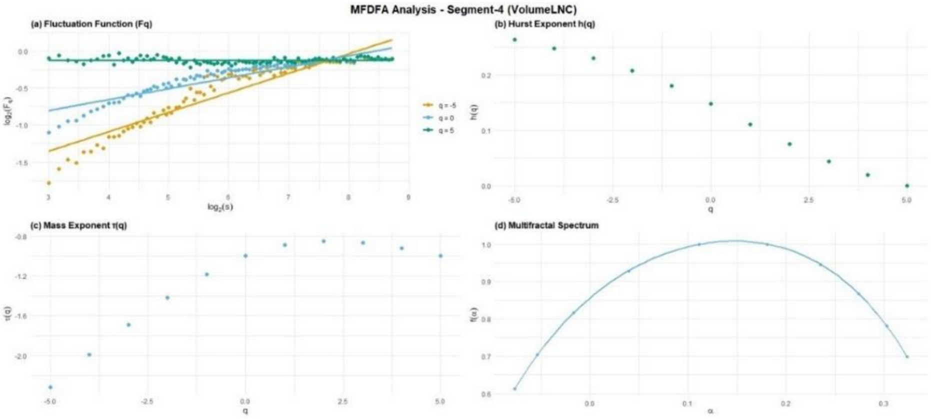
(d) - Spectrum Comparison

Figure no. 7 – Segment 3

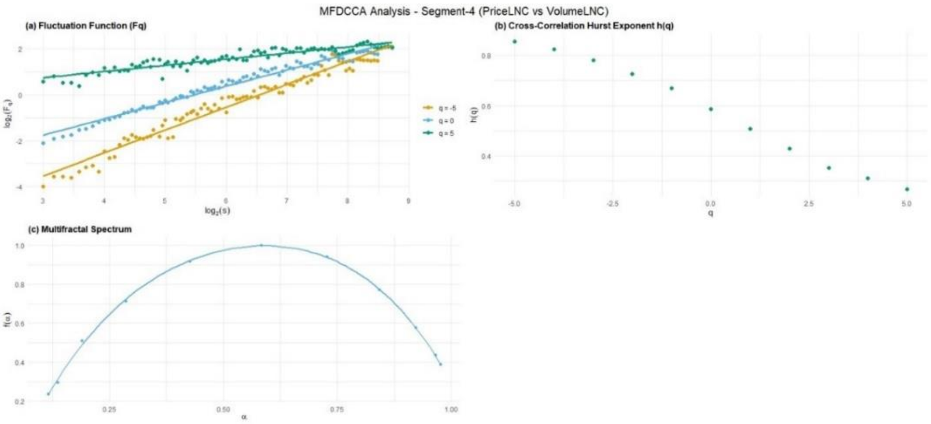
Segment 3 exhibits characteristics consistent with the post-crisis recovery phase of markets, showing partial stability in price behaviour when compared to crisis periods. As market stress begins to subside, it displays a more balanced multifractal structure, indicating a gradual decrease in volatility. However, in terms of trading volume outputs, the anti-persistent structure appears to be continuing. This indicates that there has been no clear transformation in investor behaviour. Nevertheless, a partial normalisation is observed in individual series, and the price-volume cross spectrum continues to remain broad accordingly. It also exhibits a more balanced degree of asymmetry (Figure no. 7; Table no. 3), indicating that non-linear information transmission continues beyond the crisis. The persistence of these complex co-dynamics implies that, despite the reduction in volatility pressures, non-linear information transmission continues to shape market behaviour beyond the acute crisis phase.



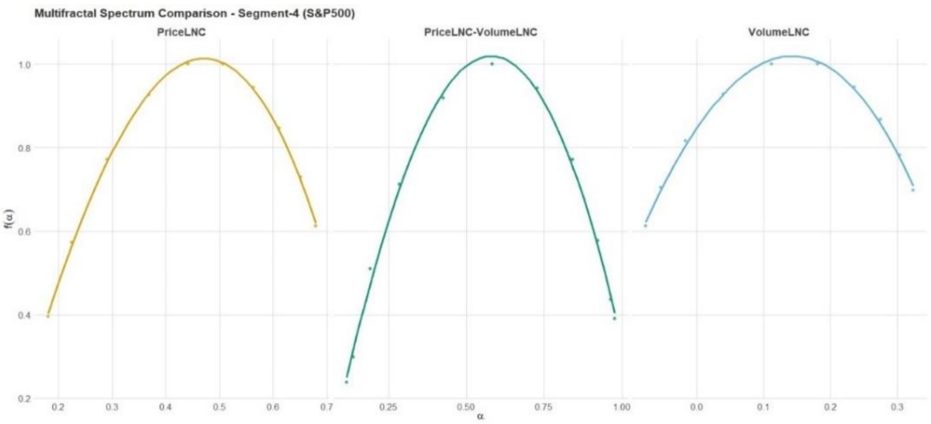
(a) - Price Spectrum



(b) - Volume Spectrum



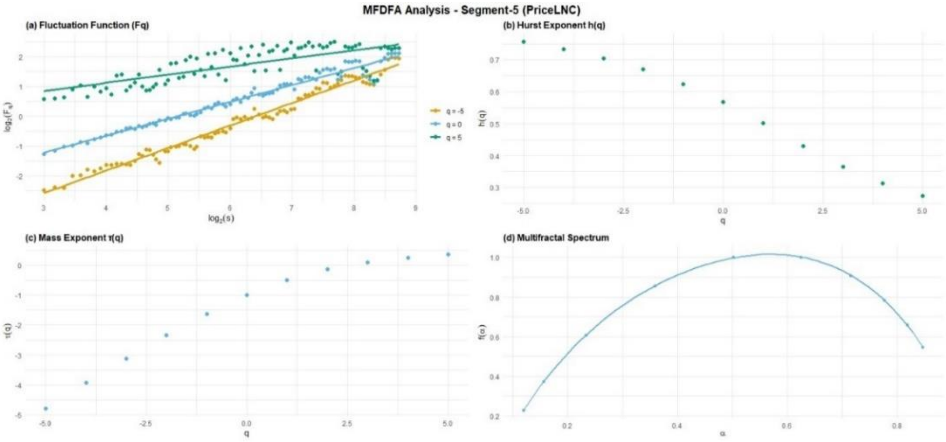
(c) - Price- Volume Cross-Correlation Spectrum



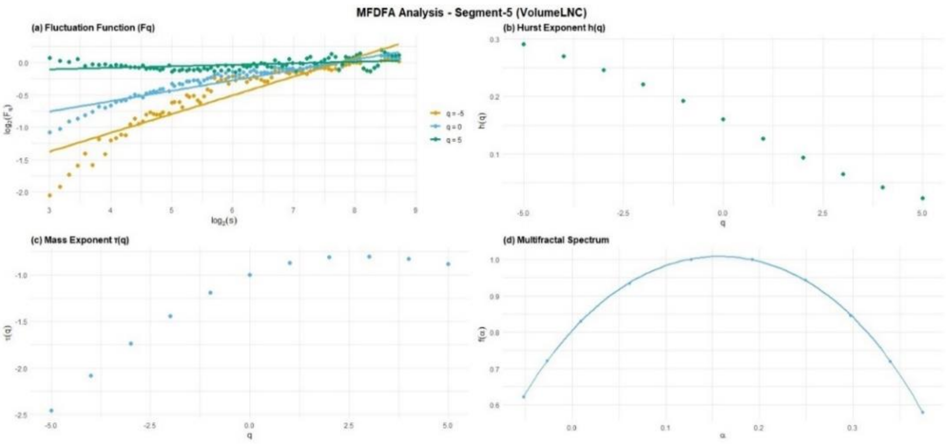
(d) - Spectrum Comparison

Figure no. 8 – Segment 4

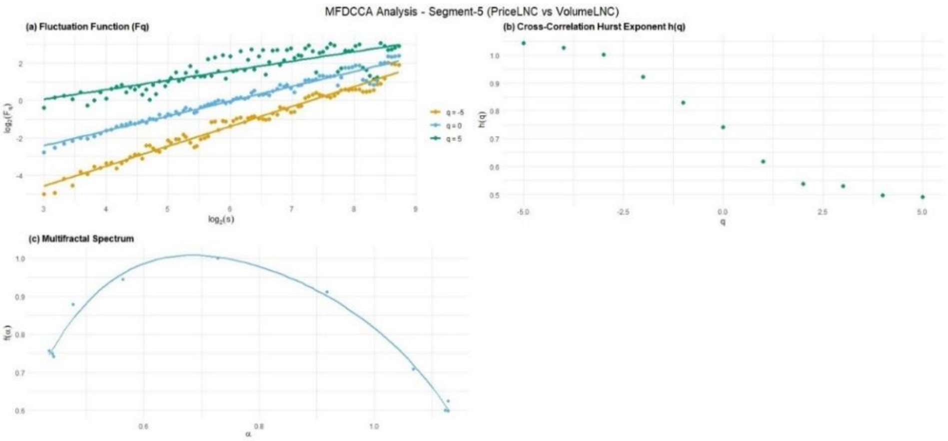
Segment 4 exhibits characteristics of a phase with market conditions consistent with low volatility. During this period, different structures are observed in small and large fluctuations, but no clear sign of extreme changes is evident, implying that pricing behaviour continues to reveal a balanced multifractal structure. Trading volume, meanwhile, remains largely anti-persistent. That is, changes are short-lived, and there is only a weak connection between past behaviour and price. Despite this, price-volume cross-correlations reveal extensive and asymmetric spectra, as shown in Figure no. 8 and summarised in Table no. 3, maintaining multifractal complexity to a non-negligible degree, despite moderate conditions in individual series. Considering all these outputs for this segment, we can state that normal financial market conditions may reduce volatility but do not eliminate non-linear dependence. These findings suggest that the dynamics embedded in the price–volume relationship may contain structurally relevant information even during periods of relative market stability.



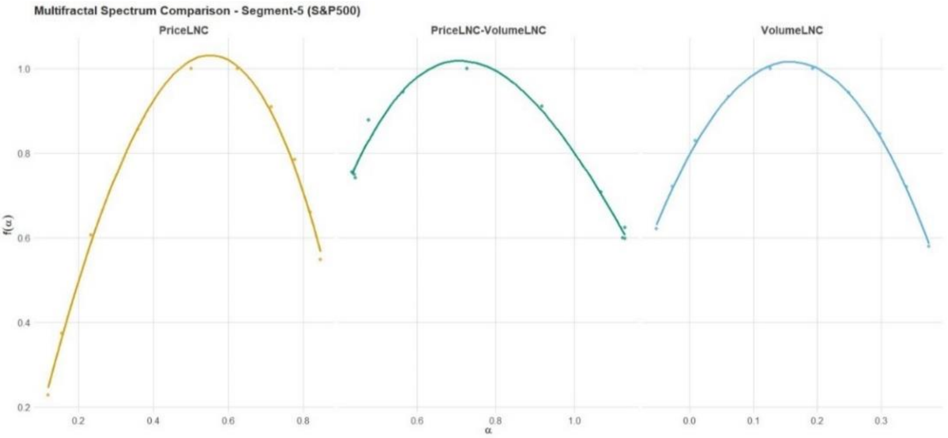
(a) - Price Spectrum



(b) - Volume Spectrum



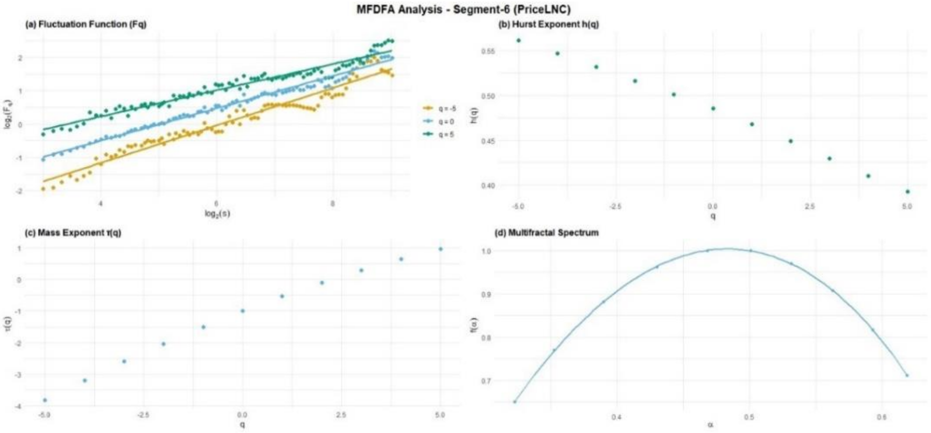
(c) - Price- Volume Cross-Correlation Spectrum



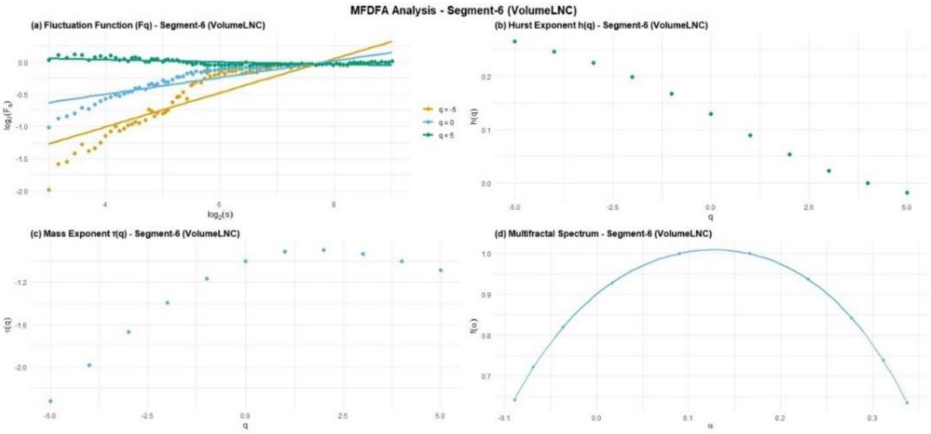
(d) - Spectrum Comparison

Figure no. 9 – Segment 5

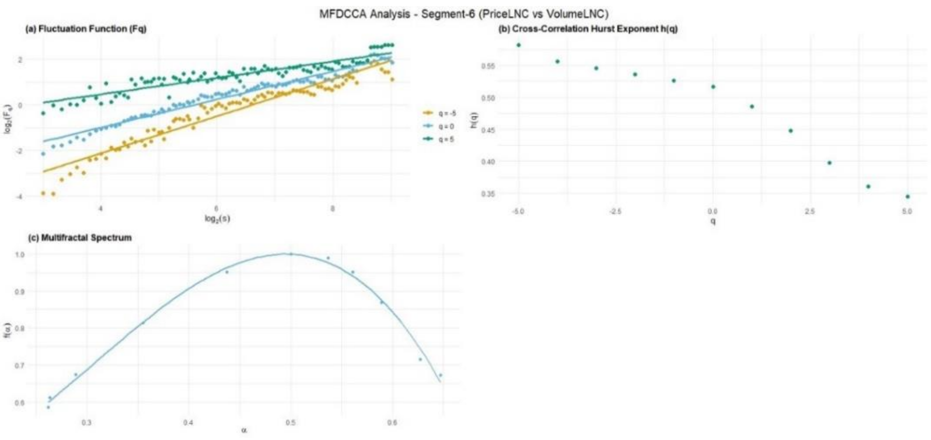
Segment 5 coincides with a period of heightened uncertainty that culminates in the Covid-19 shock and is marked by particularly strong multifractal features across market variables. Price dynamics display a rich and relatively balanced multifractal structure, pointing to the coexistence of persistent and anti-persistent behaviour across different fluctuation scales. Trading volume, by contrast, exhibits a more constrained and asymmetric multifractal profile, indicating that trading activity is largely driven by short-lived and rapidly reversing responses. The most striking feature of this segment is observed in price–volume cross-correlations, which show one of the broadest and asymmetric multifractal spectrum in the sample, as illustrated in Figure no. 9 and summarised in Table no. 3. This marked amplification of joint multifractality suggests intensified nonlinear information coupling under extreme uncertainty and highlights the heightened sensitivity of price–volume interactions to abrupt market shocks.



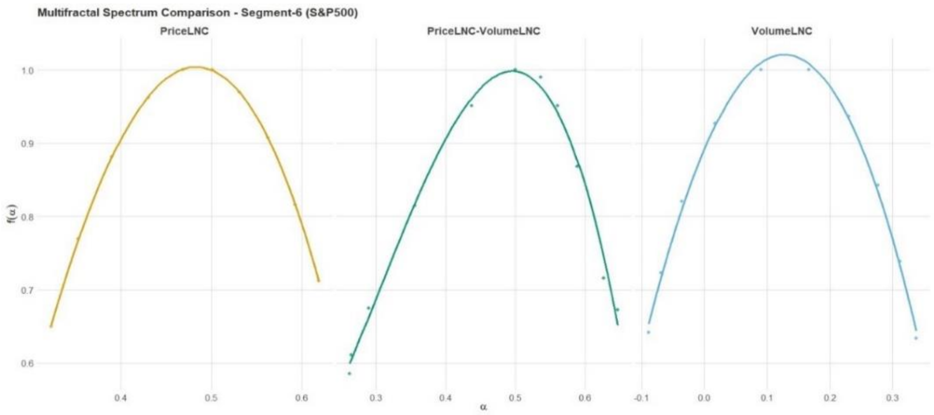
(a) - Price Spectrum



(b) - Volume Spectrum



(c) - Price- Volume Cross-Correlation Spectrum



(d) - Spectrum Comparison

Figure no. 10 – Segment 6

Segment 6 captures the post-pandemic adjustment phase and points to a moderation in multifractal intensity, albeit without a full return to linear market dynamics. Price movements continue to exhibit a clear scale-dependent structure, with different responses across small and large fluctuations and a persistent degree of heterogeneity across time scales. Trading volume, by contrast, remains largely anti-persistent, suggesting that trading activity is only weakly linked to its own past and retains a predominantly short-lived character. Price–volume cross-correlations display a multifractal structure of intermediate breadth and symmetry, more closely aligned with the price spectrum than with volume dynamics, as illustrated in Figure no. 10 and summarised in Table no. 3. Taken together, these patterns suggest that although conditions appear more consistent with market efficiency in the post-pandemic period, nonlinear joint dynamics and the structural imprint of earlier disruptions continue to influence price formation.

Table no. 3 – Multifractal spectrum characteristics by segment

Segment	Series	α_min	α_max	Δα (Width)	f(α)_max	Spectrum Shape/ Slope
1	PriceLNC	≈ 0.25	≈ 0.70	≈ 0.45	≈ 1.00	Symmetric
1	VolumeLNC	≈ -0.15	≈ 0.20	≈ 0.35	≈ 0.95	Left-skewed (anti-persistent)
1	Price-Volume	≈ 0.20	≈ 0.80	≈ 0.60	≈ 1.00	Asymmetric/ Broad
2	PriceLNC	≈ 0.20	≈ 0.70	≈ 0.50	≈ 1.00	Symmetric
2	VolumeLNC	≈ -0.10	≈ 0.25	≈ 0.35	≈ 0.95	Left-skewed
2	Price-Volume	≈ 0.15	≈ 0.85	≈ 0.70	≈ 1.00	Asymmetric/ Broad
3	PriceLNC	≈ 0.20	≈ 0.65	≈ 0.45	≈ 1.00	Symmetric
3	VolumeLNC	≈ -0.10	≈ 0.25	≈ 0.35	≈ 0.95	Left-skewed
3	Price-Volume	≈ 0.15	≈ 0.85	≈ 0.70	≈ 1.00	Asymmetric
4	PriceLNC	≈ 0.20	≈ 0.70	≈ 0.50	≈ 1.00	Symmetric
4	VolumeLNC	≈ -0.05	≈ 0.30	≈ 0.35	≈ 1.00	Slightly left-skewed
4	Price-Volume	≈ 0.20	≈ 0.95	≈ 0.75	≈ 1.00	Asymmetric/ Right-skewed
5	PriceLNC	≈ 0.15	≈ 0.85	≈ 0.70	≈ 1.00	Symmetric
5	VolumeLNC	≈ -0.05	≈ 0.35	≈ 0.40	≈ 1.00	Left-skewed
5	Price-Volume	≈ 0.10	≈ 1.00	≈ 0.90	≈ 1.00	Broad/ Asymmetric
6	PriceLNC	≈ 0.20	≈ 0.70	≈ 0.50	≈ 1.00	Symmetric
6	VolumeLNC	≈ 0.00	≈ 0.30	≈ 0.30	≈ 1.00	Left-skewed
6	Price-Volume	≈ 0.20	≈ 0.80	≈ 0.60	≈ 1.00	Symmetric/ Broad

This table summarises the multifractal spectrum characteristics of normalised log-price (PriceLNC), log-volume (VolumeLNC) and their cross-correlations (Price-Volume) over six structurally segmented periods.

5.5. Cross-segment synthesis of multifractal results

Across the six structurally identified segments, several recurring patterns emerge from the joint interpretation of the MFDFA and MFDCCA results. Price dynamics exhibit clear scale dependence throughout the sample, with persistent behaviour tending to dominate small fluctuations and anti-persistent behaviour becoming more pronounced for large fluctuations. While this scale-dependent structure is observed in every segment, the intensity and symmetry of multifractality change over time, pointing to segment-specific differences in the way prices are formed.

Trading volume follows a notably different pattern. Across all segments, volume remains largely anti-persistent and is associated with relatively narrow and left-skewed multifractal spectra. This behaviour suggests that volume dynamics are mainly driven by short-lived and small-scale trading activity, with limited long-range dependence, less directly linked to broader market conditions. Although volume series display multifractality, their structural complexity remains consistently lower than that observed for prices.

By contrast, price–volume cross-correlations stand out as the most structurally complex component of the system. In every segment, the multifractal spectra of cross-correlations are wider and more asymmetric than those of the individual price or volume series, indicating stronger nonlinear dependence and greater heterogeneity in joint dynamics. Periods of heightened uncertainty, particularly the global financial crisis and the Covid-19 shock, are associated with a marked amplification of multifractal cross-correlations, reflecting more intense interaction between prices and trading activity across scales.

The evolution of multifractal intensity also varies systematically across segments. The pre-crisis period establishes a baseline level of multifractality, while crisis and high-uncertainty phases are characterised by a pronounced strengthening of nonlinear joint dynamics. In subsequent recovery and post-crisis periods, multifractal intensity moderates but does not disappear, indicating that market behaviour does not fully revert to a homogeneous or purely random structure. This persistence of complex cross-dynamics, even under relatively stable conditions, suggests that market efficiency adjusts over time rather than remaining fixed.

Overall, the cross-segment evidence indicates that analyses focusing on prices or trading volume in isolation provide only a partial view of market dynamics. A joint multifractal perspective on price–volume interactions captures segment-specific patterns of information transmission more effectively and highlights the time-varying nature of market efficiency across different scales, consistent with the Adaptive Market Hypothesis and the Fractal Market Hypothesis.

Taken together, the empirical findings offer coherent answers to the research questions outlined in the Introduction. Both price and trading volume series exhibit multifractal and scale-dependent behaviour across all segments identified by endogenous structural breaks, addressing the first research question. Price–volume cross-correlations, in turn, differ systematically from individual series by displaying stronger nonlinearity and higher structural heterogeneity, which speaks directly to the second research question. Finally, the segment-specific evolution of multifractal properties points to time-varying departures from weak-form market efficiency rather than a permanent breakdown, lending empirical support to the Adaptive Market Hypothesis and the Fractal Market Hypothesis.

6. CONCLUSION

This study offers a comprehensive examination of the multifractal behaviour of prices, trading volume, and their cross-correlations in the S&P 500 index by jointly applying MFDFA and MFDCCA within a structural break framework. By considering both the full sample and six endogenously identified segments, the analysis shows that market dynamics are scale-dependent, multifractal, and subject to change over time. Rather than pointing to a permanent breakdown of market efficiency, the findings indicate segment-specific departures from weak-form efficiency, reflected in persistent and heterogeneous multifractal structures. These patterns are consistent with the Adaptive Market Hypothesis and the Fractal Market Hypothesis, suggesting that market efficiency adjusts in response to evolving economic conditions, investor behaviour, and information flows. Consistent with the cross-segment synthesis, the evidence further shows that multifractality and scale-dependent behaviour persist across all segments, although with varying intensity. While price and trading volume display distinct structural features, their joint dynamics consistently exhibit greater heterogeneity and stronger nonlinear dependence, particularly during periods of elevated uncertainty.

The analyses indicate that both price and volume series exhibit a multifractal structure. The Hurst exponent $h(q)$, the nonlinear mass function $\tau(q)$ and the expanding multifractal spectrum $f(\alpha)$, which varies with the degree of moment q, show that the series do not have a fixed fractal structure and exhibit multifractal properties. In particular, the fluctuation functions corresponding to low and high q values show that small and large-scale fluctuations exhibit different behaviours at different time scales; this situation reveals the presence of multifractal behaviour of statistical properties in the series.

Segment-based analyses show that the S&P 500 time series exhibits a time-varying multifractal structure. Each segment has different spectrum widths, slopes and maximum points. This is thought to indicate that time-varying dynamics in the market, differences in investor behaviour and reactions to external shocks affect the structure of the series. The asymmetric structure observed in the spectral profiles of the volume series and the fluctuations in the spectrum width suggest that these series may exhibit diffuse multifractal features depending on time.

MFDCCA analyses have been very useful in examining the common multifractal behaviour between price and volume series. The wider and more asymmetrically distributed cross-spectra suggest that price-volume interactions have stronger and more spectrally distinct multifractal features compared to individual series. These results suggest that volume is not a passive indicator but an important component of price formation, especially in market structure studies.

In the full-sample MFDFA analysis, raw log-return and log-volume change series (PriceLNC and VolumeLNC) were used without normalization to preserve the natural multifractal structure. However, all cross-correlation (MFDCCA) and segment-wise MFDFA/MFDCCA analyses were conducted using normalized series to ensure consistency and comparability across scales. In the calculation of fluctuation functions, the squaring approach commonly used in the literature was adopted. This method, while providing theoretical continuity, also allowed for a more stable calculation of $\tau(q)$, $h(q)$, α and $f(\alpha)$ functions.

As a result, the S&P 500 index exhibits a scale-dependent, multifractal and time-varying structure in terms of both price and volume. This structure does not suggest a permanent violation of the weak-form Efficient Market Hypothesis. Instead, it calls into question the strict random walk assumption at certain market segments and time scales, pointing to segment-dependent persistence, memory effects, and heterogeneous behaviour. The patterns observed across segments further indicate that market efficiency evolves over time and varies across scales, rather than remaining uniform throughout the sample period. The detection of such complex structures once again reveals the importance of multifractal approaches in financial modeling, forecasting, and risk management processes.

The present findings are consistent with the conclusions of some prior studies, particularly those by Kantelhardt *et al.* (2002) and Zhou (2008), which suggest that financial time series have multifractal properties. The apparent multifractality in price series is similarly found by Miloş *et al.* (2020) and Carbone *et al.* (2004). Our results showing that volume series also exhibit multifractal structures are in line with the approaches emphasised in studies such as Jizba *et al.* (2012) and Wątorek *et al.* (2021) that market volume is a reflection of information flow and investor reactions. Moreover, the existence of cross-multifractal relations between price and volume in the context of MFDCCA results is also revealed in studies such as Oświęcimka *et al.* (2006) and Wang *et al.* (2013). However, the asymmetric structure of the spectrum, especially in the volume series, differs from some related findings in the literature, such as Rak *et al.* (2015), as well as earlier studies on financial correlation structures (Drożdż *et al.*, 2001). In conclusion, this study is consistent with the existing literature in terms of both its methodological approach and the results obtained, and it is thought that this study will contribute to the literature by presenting original findings in some aspects.

6.1. Limitations and Future Research

Despite the robustness of the empirical findings, several limitations should be acknowledged. First, the analysis focuses on a single benchmark index, which may limit the generalisability of the results to other markets or asset classes. Second, although the sample period is relatively long, multifractal outcomes can be sensitive to the choice of scale ranges and segment lengths inherent in the methodology. Third, methodological constraints associated with MFDCCA – such as the emergence of negative fluctuation functions for certain moments – necessitated the use of a second-moment-based approach, which may restrict the interpretation of higher-order dynamics. In addition, during the implementation of MFDFA, occasional negative values of the fluctuation function $F_q(s)$ for certain moment orders required the adoption of a moment-restricted formulation, which, while ensuring numerical stability, may lead to a less sharp representation of the underlying multifractal structure.

From a scientific perspective, this study contributes to the literature by highlighting the importance of jointly analysing prices, trading volume, and their multifractal cross-correlations within a regime-dependent framework. By showing that nonlinear price–volume interactions exhibit stronger and more heterogeneous multifractal properties than individual series, the results provide new insights into the mechanisms of information transmission and the time-varying nature of market efficiency.

From a practical standpoint, the results carry several important implications. For regulators, the pronounced strengthening of price–volume cross-dynamics during turbulent periods suggests that monitoring joint multifractal indicators may help identify early signals of market stress and instability. For investors and portfolio managers, the scale-dependent and segment-specific nature of market efficiency implies that risk–return characteristics vary across time horizons, underscoring the potential value of adaptive and regime-aware investment strategies. For risk managers, the broader and more asymmetric multifractal spectra observed in price–volume cross-correlations highlight the importance of accounting for nonlinear dependence structures that extend beyond standard linear correlation measures, particularly in periods of heightened uncertainty. In this context, understanding the nature of the market allows the use of volatility clustering as a tool to measure and forecast risk, incorporating long-term price dependence and the tendency of adverse shocks to cluster. Once market characteristics are properly identified, risk forecasting models can be enhanced (Kobeissi, 2013).

Researchers who will conduct future studies that will contribute to the literature may be advised to use MFDFA and MFDCCA analyses together in order to identify the multifractal structure and to take volume series into account. In addition to this study with the S&P 500 index, which reflects a significant economic size, similar studies for the indices of other major stock exchanges and indices representing relatively smaller economies can increase the depth and diversity of the literature. Thus, the change in efficiency over time and the possible different structures of self-similarity and long memory characteristics can be revealed. A technical recommendation for future researchers on the subject is to choose the values such as series size and scale range carefully when running these analyses. Especially in the MFDCCA method, it would be useful to make a meticulous study on the method to be preferred to avoid negative *F(q)* values. Future research could build on the present framework by exploring alternative multifractal approaches, such as cross-multifractal spectrum methods, to further examine the robustness of price–volume interactions across different market segments.

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