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IFRS 13: What Certainty Equivalent Might be Requested when Deriving a Fair Value Based on Risk-Adjusted Expected Cash Flows?

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Abstract: The study raises the point to elicit thresholds for certainty equivalents when determining the fair value using Method 1of the present value techniques within the methodology of income approaches. Through applying the risk-measure Value at Risk as indicator for certainty equivalents, it becomes possible to utilise the experience gained from risk management practice. Based on the calculation of certainty equivalents (the risk-adjusted expected income and expenses) observable in AAA-, Baa- and high-yield-rated U.S. corporate bonds, the corresponding Values at Risk were assessed by modelling different probability distributions. The studies reveal that investors in U.S. corporate bonds had accepted certainty equivalents that approximately correspond to Values at Risk with a confidence level in the range between 50 and 75% when taking the yield premium as criterion. In risk management practice, Values at Risk with confidence levels of above 80% are recommended. However, the safety margins then to be demanded reach values of approx. 17-25% on the expected value, which is in drastic contrast to the historical certainty equivalent coefficients

Keywords: fair value; IFRS 13; certainty equivalent method; three-point scenarios; probability assessment.

JEL classification: M41; C18; C30; C44.

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1. INTRODUCTION

International Financial Reporting Standard 13 Fair Value Measurement (IFRS Foundation, 2022b) is applied for the measurement of a specific asset or a specific liability (IFRS Foundation, 2012). It applies to both initial and subsequent measurement if fair value is required or permitted by other IFRSs (IFRS13.8). IFRS defines fair value as the price that would be received to sell an asset or paid to transfer a liability in an orderly transaction between market participants at the measurement date under current market conditions (Özerhan and Sultanoğlu, 2017; IFRS Foundation, 2022a, 2022b; Hall *et al.*, 2023). This approach stands in contrast to an approach that focuses on historical costs (Lennard, 2018; Gilliam and Hofmann, 2018a, 2018b).

From the three valuation techniques in IFRS 13 to derive fair value (market approach, cost approach and income approach; IFRS13.61 to IFRS13.66), risk-adjusted expected cash flows are inputs to one of two methods within present value techniques which belong to the income approach (IFRS13.B23).

Method 1 within the income approach (IFRS 13.B25), the "certainty equivalent method" (Dayananda *et al.*, 2002), follows the utility theory paradigm of replacing possible cash flows (resp. income and expenses, following IFRS13.B10), which are uncertain, with a monetary value that reflects the (market participants') utility of these uncertain cash flows. Risk is removed by substituting possible, but uncertain cash flows with its certainty equivalent, i.e., the monetary valued utility of the uncertain outcomes. Formally, a 'cash risk premium' is subtracted from the expected value of possible future cash flows ('expected cash flows' in IFRS 13) leading to 'risk-adjusted expected cash flows', the certainty equivalent. In that case the market participant would be indifferent as to the asset held. As the certainty equivalent covers the systematic (i.e., market) risk and market participants are not compensated for the risk specific to a particular asset or liability, which is diversifiable ('unsystematic risk') (IFRS13.B24), the fair value is calculated by discounting the risk-adjusted expected cash flows at a risk-free interest rate.

Method 2 of the fair value measurement within the income approach, the "risk-adjusted discount rate method" (Dayananda *et al.*, 2002), uses the expected value of possible future cash flows, derived from market perspective, which are not risk-adjusted, and a discount rate which is adjusted to include the risk premium that market participants require (IFRS 13.B17, IFRS 13.B26). When deriving the expected value an entity may begin with its own data, but it shall adjust those data if 'reasonably available' information indicates that other market participants would use different data (IFRS 13.87-89). An entity shall take into account all information about market participant assumptions that is 'reasonably available'. To gain the fair value, the expected cash flows are discounted at a rate which incorporates a risk premium to the risk-free interest rate (further: 'risk-adjusted interest rate').

IFRS13.B36 mentions the possibility to derive possible future cash flows (resp. income and expenses; IFRS13.B10) from unobservable inputs when evaluating a cash-generating unit or a decommissioning liability assumed in a business combination. If derived from unobservable inputs (like projected cash flows), the measurement of fair value belongs to 'Level 3 inputs', the least prioritised category (Rohleder *et al.*, 2017). The highest priority should be given to quoted prices in active markets for identical assets ('Level 1 inputs'; IFRS 13.76), followed by including observable inputs other than quoted in Level 1 inputs ('Level 2 inputs'; IFRS 13.82-83).

The present value technique's significance, especially for Method 2 (risk-adjusted discount rate method), can be assessed following the exemplifications in IFRS 13.IE35-IFRS 13.IE39, as well as reports from the literature (i.e., for combining businesses: IFRS Foundation (2022a); for cash generating units: BDO (2022), EPRA (2013) and IPSAS (2024), for assets used in real estate industry or for investments in property under construction: (IFRS Foundation, 2012; Busso, 2014; Sundgren *et al.*, 2018; Baur *et al.*, 2025).

The applicability of Method 1 (certainty equivalent method) mainly suffers from comparative data and a methodology to derive certainty equivalents (from the market perspective) (Dayananda *et al.*, 2002; KPMG, 2011).

This research is an attempt to contribute to the applicability of Method 1.

This study focuses first on the aspect which certainty equivalent could be accepted by an auditor, based on historic data, if an entity follows Method 1 instead of applying Method 2. This could result in formulating an expectation in cases of applying Method 1.

Derived are historic certainty equivalents from historic bond yield premiums under the assumptions that bond yields represent the interest rate used by market participants to find the net present value (IFRS Foundation, 2022b) and that the identity of the net present values when applying Method 1 or Method 2 of the expected present value techniques holds (Beccacece *et al.*, 2018; IFRS Foundation, 2022b).

Secondly, the paper raises the question as to which confidence level could be assigned to the derived historical certainty equivalents if the risk-measure Value at Risk is used as an operationalisation of the certainty equivalents. If this is successful, especially if the historical data leads to confidence levels that correspond to the conventions in risk management, then these results could enable better traceability and also better control of the statement, since not only the probability function as in Method 2 must be specified for the derivation of the Value at Risk, but also the confidence level on which the decision is based.

The certainty equivalent of possible but uncertain outcomes gives their utility (Bruner *et al.*, 1998; Fabozzi *et al.*, 2007; Crundwell, 2008; Zhang, 2010; Pagani, 2015). In risk management practice, a simplification is applied by measuring the utility in units of the target variable (e.g. loss, earnings) – as it is done in IFRS 13.B26.

In risk management, the usage of the expected value (the probability weighted sum of possible outcomes) as decision criterion is disputed even if one takes the view that the expected value can approximate the median (the value that will be exceeded by higher numerical values with a cumulative probability of 50% or less).

Empirical studies recommend that both, the expected value and the median, should be disregarded as a decision criterion (or, here, as a decision maker's certainty equivalent) unless a high number of comparable situations with very similar probability distribution exist or are created. These studies originate from the following areas: project budgeting (Flyvbjerg, 2007; Bodea and Purnus, 2012; Garvey *et al.*, 2012; Lee *et al.*, 2012; Thomas and Fitch, 2014) and assessing risk positions within a financial institution (Cruz, 2006; Gaudin, 2016; Pfeifer and Ragulina, 2018).

The same consideration can be found in studies evaluating the benefit of an investment by assessing net cash flows (Frey and Rubin, 1992; Kremers, 2002; Beisler, 2011; Hill, 2012; Gleißner *et al.*, 2021).

To illustrate this criticism: The expected value gives, in case of symmetrically distributed outcomes, the value which will be exceeded by the group of outcomes with higher numerical values or undercut by the group of outcomes with lower numerical values with a probability of approximately 50% (Jorion, 2007; Huschens, 2017; Klugman *et al.*, 2019). Taken a situation with six equal probable outcomes, as it is when throwing a fair dice, the expected value of 3.5 (say, the fair value) will be undercut by the lower numerical outcomes of one, two and three with a cumulative probability of 50%. Further, the expected value of 3.5 will be exceeded by the higher numerical outcomes of four, five and six with a cumulative probability of 50%. The same applies to the median. In the given example, the median is equal to three, which means that higher numerical values exceed this value with a probability of less than 50% and that lower values occur with a probability of less than 50% (when excluding the probability of the median value itself).

This means, when basing a business decision on the median (or on the expected value under the assumption of a symmetrical probability distribution), the confidence in gaining the median value when implementing the decision can solely be labelled as 'more likely than not' (Bohušová *et al.*, 2014) or 'about as likely as not' (Mastrandrea *et al.*, 2010), since the probability of falling below or exceeding this value is about 50%.

For gaining a higher confidence level, the risk-measure Value at Risk is recommended (Jørgensen and Teigen, 2002; Jorion, 2007; GAO, 2009; Weapon Systems Acquisition Reform Act, 2009; Huschens, 2017; Klugman *et al.*, 2019). The risk-measure Value at Risk gives the value that is outperformed by more favourable (!) values with the probability of the selected confidence level. The Value at Risk with a confidence level of 90% (VaR90) gives the value that will be outperformed by more favourable values with a cumulative probability of (at least) 90%.

Formally, the utility of the possible but uncertain outcomes is derived by solely regarding the most unfavourable outcome of the group of outcomes which is defined by their cumulative probability of occurrence (determined by the selected confidence level), if all outcomes are considered in ascending order of advantage. No utility is ascribed to the outcomes which are more favourable and which are less favourable than the Value at Risk.

To illustrate: When selecting a VaR90 as certainty equivalent, no utility is attributed to more favourable outcomes that occur with a cumulative probability of less than 0.90, nor to less favourable outcomes that occur with the complementary probability of the confidence level (here: less than 0.10).

For deriving the Value at Risk, the probability distribution of possible future outcomes has to be assed (congruent to the exemplifications in IFRS Foundation (2022b) from B28 to IE63) and, based on it, its cumulative probability distribution has to be calculated. The percentiles of the cumulative probability distribution are the basis for deriving the Value at Risk. When applying the measure to cash flows resp. to income and expenses, then higher numerical values stand for more favourable outcomes. Therefore, the Value at Risk with a defined certainty level can (approximately) be derived from the percentile that is identical with the complementary probability of the certainty level (Jorion, 2007; Huschens, 2017; Klugman *et al.*, 2019). In case of VaR90, the percentile 10 of the cumulative probability distribution approximates the value which will be exceeded by more favourable values with a cumulative probability of 90%.

If a high number of situations with a very similar probability distribution of the possible future outcomes exists or is created, the median (the Value at Risk with a confidence level of 50% [VaR50]) is seen as acceptable in risk management practice (Flyvbjerg, 2004). However, if only a small number of projects with a similar probability distribution exists or is created (this could be the case when combining businesses or be the cases of evaluating assets in real estate industry or investments in property under construction) higher confidence levels are proposed: between 80 to 90 % in project budgeting (Jørgensen and Teigen, 2002; Flyvbjerg, 2004; GAO, 2009; Weapon Systems Acquisition Reform Act, 2009) or between 95 to 99% for the valuation of risk positions within a financial institution (Hendricks, 1996; Jorion, 2007).

The following Section 2 focuses on the first aspect of this study: which certainty equivalent could be accepted by an auditor, based on historic data. It gives the mathematical basis of deriving certainty equivalents from the identity of the net present values when applying Method 1 or Method 2 as well as the results of applying this methodology to historic AAA- and Baa-rated U.S. corporate bond yields.

Section 3 and Section 4 deal with the second aspect: the question of which confidence level can be assigned to the derived historical certainty equivalents if the risk-measure Value at Risk is used as an operationalisation of the certainty equivalents.

Section 5 focuses on an analysis and discussion of the results gained. Within that section two further analyses are reported to check the plausibility of the results. Section 6 discusses the results and give explanations for the results found. Section 7 summarises the results and gives the author's conclusions regarding the applicability of Method 1 of the present value techniques within the income approach.

This study contributes to the theoretical discussion on the applicability of Method 1 of the present value techniques within the income approach. Further, this paper gives an overview of how to derive the risk-measure Value at Risk from probabilistic three-point scenarios based on the illustrative example in IFRS Foundation (2022b) B25.

2. DERIVING CERTAINTY EQUIVALENTS FROM HISTORIC BOND YIELD PREMIUMS

In order to assess which minimum certainty equivalents should be required when deriving a fair value using Method 1 of the expected present value techniques, the identity of the net present values when applying Method 1 and when applying Method 2 is utilised (Dayananda *et al.*, 2002; Beccacece *et al.*, 2018; IFRS Foundation, 2022b).

This relationship is used to calculate the certainty equivalents of historic bond yield premiums of Moody-AAA- and Moody-Baa-rated U.S. corporate bond yields in relation to market yields on U.S. treasury securities at 10-year constant maturity. The analysis is based on the assumption that bond yields represent the interest rate used by market participants to determine the net present value (IFRS13.B14).

Method 1 adjusts the expected cash flows of an asset for systematic (i.e., market) risks by subtracting an amount (the 'cash risk premium' in IFRS) from the expected value ('expected cash flows' in IFRS), to come to the certainty equivalent ('risk-adjusted expected cash flows' in IFRS 13). In Method 2, the expected value is discounted with an interest rate, which contains an interest rate risk premium on the risk-free interest rate that market participants demand when these expected cash flows are offered to them ('risk-adjusted interest rate').

The certainty equivalent (regarding a period) can be calculated from this as follows:

Let

CE / (1 + r_rf): Net present value of the certainty equivalent

EV / (1 + r_rf + r_rp): Net present value of the expected value

with:
CE: Certainty equivalent ('risk-adjusted expected cash flows')
EV: Expected value ('expected cash flows')
r_rf: Risk-free interest rate
r_rp: Interest rate premium

Let:
r_ra: Risk-adjusted interest rate

With:

$$r\_ra = (r\_rf + r\_rp) \quad (1)$$

From:

$$CE / (1+ r\_rf) = EV / (1 + r\_rf + r\_rp) \quad (2)$$

follows

$$CE / (1+ r\_rf) = EV / (1 + r\_ra) \quad (3)$$

$$<=> CE = EV * (1+ r\_rf) / (1 + r\_ra) \quad (4)$$

To rule out any influence from the numerical values of the certainty equivalent and the expected value, the certainty equivalent coefficient (CEC) is used as a reference standard: the ratio of the certainty equivalent to the expected value (Dayananda *et al.*, 2002; Crundwell, 2008; Zhang, 2010).

A certainty equivalent coefficient reaches the value of one if the certainty equivalent is identical with the expected value: a market participant regards the possible (uncertain) cash flows as risky as the corresponding risk-free cash flows. A certainty equivalent coefficient of 0.9 is to be interpreted as meaning that the certainty equivalent corresponds to 0.9 times the expected value or that a safety margin of 10% is applied on the expected value. A certainty equivalent coefficient cannot achieve values above 1, as this would mean that the possible (uncertain) cash flows are considered less risky than the corresponding risk-free cash flows.

The certainty equivalent coefficient is calculated as follows:

Let

CEC: Certainty equivalent coefficient

and

$$CEC = CE / EV \quad (5)$$

Then follows from (3):

$$\begin{aligned} CE / (1+ r\_rf) &= EV / (1 + r\_ra) \\ <=> CE / EV &= (1 + r\_rf) / (1 + r\_ra) \\ <=> CEC = CE / EV &= (1 + r\_rf) / (1 + r\_ra) \end{aligned} \quad (6)$$

The historic certainty equivalent coefficients are calculated by formula (6). Chosen are 9659 data points of Moody-AAA- (FRED, 2025a) and 9659 data points of Moody-Baa-rated U.S. corporate bond yields (FRED, 2025b) from 1986-01-02 to 2024-08-15, representing risk-adjusted interest rates. For the analysis, these data are set in relation to market yields on U.S.

treasury securities at 10-year constant maturity (FRED, 2025c) representing risk-free interest rates, similar Dayananda *et al.* (2002); BDO (2022). The certainty equivalent coefficients are derived by the author per data point on daily basis to eliminate seasonal effects, following the methodology of FRED (2025d, 2025e).

The 19,318 data points are regarded together and condensed in 11 classes with equal width intervals. Table no. 1 gives the group average per class, and its relative frequency. Location parameters are given in Table no. A1 in Annex 1. A regression analysis of the time series shows a negligible influence over time (slope: -4E-07; coefficient of determination r^2: 0.051) so that the data can be regarded as representative (see Figure no. A1 in Annex 1).

Class 1, the class with the highest certainty equivalent coefficients (group average: 0.9932, lower group bound 0.9923), does only include AAA bonds. For that reason, this class is selected for further analyses.

The weighted average of classes 9 to 11 (0.9494) is selected as representative of very low certainty equivalent coefficients, as only Baa-rated bonds in a recessionary phase (following the definition of FRED (2025a, 2025b)) are represented in these three classes.

The classification of the weighted average of the classes 9 to 11 as 'very low certainty equivalence coefficients' is justified by the fact that the certainty equivalence coefficient of less than 0.9661 (class 7) are 1.5-times the interquartile range below the first quartile. The definition of an outlier follows the convention, that values laying 1.5-times the interquartile range below the first quartile or laying 1.5-times the interquartile range above the third quartile can be classified as outliers (Cooper and Schindler, 2003; Anderson *et al.*, 2007; Howell, 2010).

Table no. 1 – Certainty equivalents coefficients for Moody-AAA- and Moody-Baa-rated bond yields to U.S. treasury securities at 10-year constant maturity (1986-01-02 to 2024-08-15)

	CEC (group average)	Relative frequency
Class 1	0.9932	0.0724
Class 2	0.9900	0.1974
Class 3	0.9848	0.3113
Class 4	0.9803	0.2281
Class 5	0.9751	0.1074
Class 6	0.9705	0.0652
Class 7	0.9662	0.0089
Class 8	0.9610	0.0014
Class 9	0.9553	0.0011
Class 10	0.9503	0.0039
Class 11	0.9459	0.0030

3. DERIVING VALUES AT RISK FROM ASSESSING PROBABILITIES TO THREE-POINT SCENARIOS

The risk-measure Value at Risk is gained from assessing the probability distribution of possible future outcomes and from calculating its cumulative probability distribution.

Examples of how to come to a probability distribution can be derived from the illustrations of the disclosure requirement (IFRS13.IE63) and IFRS13.B17, IFRS13.B27. Following these illustrations, the range of the possible future outcomes (cash flows resp. income and expenses, IFRS13.B10) is derived from assessing the range of the influencing factors which leads to an assessment of the minimum, the maximum and the most-likely outcome.

This approach corresponds to the technique of creating (probabilistic) three-point scenarios (Schnaars and Ziamou, 2001; Chermack and Coons, 2012). Within that research field, a solution for the situation can be found if many factors influence the outcome, too. Then, the three scenarios are derived from assuming (a) all influencing factors would reach the unfavourable value (leading to the worst-case scenario), (b) all would reach the most-likely value (most-likely-case scenario), and (c) all influencing factors would reach the favourable value (leading to the best-case scenario) (Purnus and Bodea, 2013).

After recognising these three possible outcomes, the corresponding probability of occurrence is determined. The probabilities of the three scenarios can be assessed directly (as in the example of IFRS13.B27) or indirectly (excellent reviews on the fulfilment of this task can be found at Stael Von Holstein and Matheson (1978); Garthwaite *et al.* (2005); Elfadaly (2012); Grigore *et al.* (2013); Goodwin and Wright (2014); Zondervan-Zwijnenburg *et al.* (2017). It should be mentioned that in order to determine the probability distribution on the basis of the three-scenario approach, it is advisable to estimate the probability of occurrence for two further points: for the value between the worst-case value and the most-likely value as well as for the value between the most-likely value and the best-case value (Kasprik, 2024).

Practical applications of deriving the fair value from probabilistic scenarios are reported in BDO (2022); IFRS Foundation (2022a); Škoda *et al.* (2023); Baur *et al.* (2025).

For illustration, the situation described in IFRS13.B25 is taken as example. It is assumed that the possible future cash flows are those of a cash-generating unit with a useful lifetime of four years. It is also assumed that the cash flows describe the average (and, therefore, identical) situation in each period over these four years. The data are derived from assessing the consequences of two influencing factors (e.g. rental value and occupancy rate in a real estate property portfolio) with three states (unfavourable, most-likely, favourable). Following the multi-factor three-scenario approach (see above), three scenarios are derived: (a) worst case: Currency Units (CU) CU500, (b) most-likely case: CU800, (c) best case: CU900. In a further assessment, the probabilities of the scenarios are derived by the entity's knowledge of the market: (a) worst case: point probability of 0.15, (b) most-likely case: point probability of 0.60, (c) best case: point probability of 0.25.

The probability assessment has to include the probability of the intermediate outcomes because otherwise distinguishable Values at Risk cannot be derived and also because of the difficult-to-accept assumption that no further outcomes are to be expected between the point estimates (working with three discrete scenarios means, in terms of probability theory, that no further events exist in the event space).

The author considers here two techniques for evaluating the probabilities of the outcomes between the point estimates. A first technique follows the view that the point estimates give the midpoint of an interval (Anderson *et al.*, 2007) and the assessed point probability is equal to the interval probability.

In that case, the interval width is derived from the smallest distance of the values gained from the scenarios: here CU100 (the difference between the best-case and the most-likely-case scenario). From this follows, that the most-likely-case scenario covers the values between CU750 and CU850 (with an interval midpoint of CU800), the best-case scenario values between CU850 and CU950 (with an interval midpoint of CU900). The worst-case point estimate (CU500), taken as midpoint, covers 2.5 intervals to the lower bound of the most-likely interval (CU750), therefore, it covers in total five intervals (the range between CU250 and CU750 with the midpoints CU300, CU400, CU500, CU600 and CU700). This means (under the assumption of equal distributed class member), the assessed probability of 0.15 has to be shared on each interval, leading to a probability of 0.03 for each of the five intervals. Figure no. 1 gives the probability distribution when dividing the bandwidth in 693 equal width intervals in order to derive discriminating Values at Risk. The corresponding cumulative probability distribution is given in Figure no. 2.

For deriving the net present value, the confidence level required by market participants must be determined for each year. An increasing confidence level over the years corresponds to the increasing uncertainty in the assessment of possible conditions in the more distant future (Dayananda *et al.*, 2002).

Assuming that market participants are expected to select a confidence level of 80% (percentile 20, VaR80), for the second a VaR85 (percentile 15), the third a VaR86 (percentile 14) and the fourth a VaR87 (percentile 13), then the net present value is reached by discounting the cash flow of CU757.6 (first year), CU749.5 (second year), CU716.2 (third year) and CU682.8 (fourth year) with the (constant) risk free interest rate of 0.05. The net present value is CU2,583 over the four periods (when disregarding capital outlay).

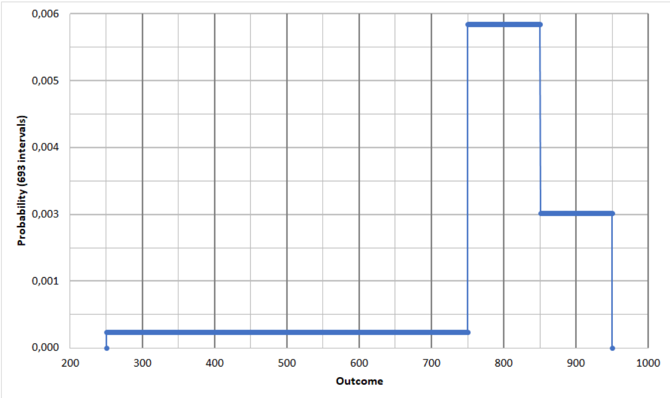

Figure no. 1 – Probability distribution (technique 1)

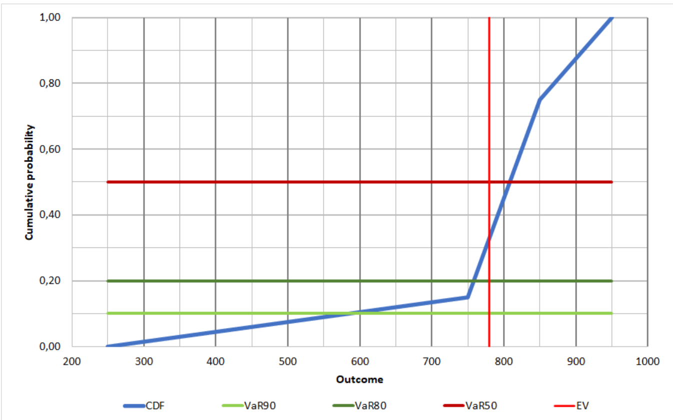

Figure no. 2 – Cumulative probability distribution (technique 1)

A second technique follows the view that the probabilities between the reference points (here: the point estimates of the three scenarios) follow the estimates in proportional sequence ('relative likelihood technique'). The estimates are regarded as likelihood assessments which fix the likelihood ratios between the reference points. The likelihood ratios of the intermediate values are gained by a linear interpolation between two likelihood assessments (Spetzler and Stael von Holstein, 1975; Stael Von Holstein and Matheson, 1978; Bonano *et al.*, 1990; Garthwaite *et al.*, 2005; Goodwin and Wright, 2014).

For implementing this technique, the bandwidth is divided in a high number of equal width intervals (here: 693 intervals) through which a granular calculation of likelihood ratios becomes possible. The probability distribution is gained by normalising the likelihood ratios to reach values between zero and one (Ludke *et al.*, 1977; Bonano *et al.*, 1990).

For the example in IFRS13.B25, the absolute lower limit is set (following the rationale described in technique 1) to CU250. The absolute upper limit is set to CU1354 in order to reach the identical expected value of CU780 as in IFRS13.B27. Four linear interpolations are realized: (a) between the absolute lower limit (CU250; likelihood ratio: 0) and the worst-case point estimate (CU500; likelihood ratio: 0.15), (b) the worst-case point estimate and the best-case estimate (CU800; likelihood ratio: 0.6), (c) the best-case estimate and the worst-case estimate (CU800; likelihood ratio: 0.25), (d) worst-case estimate and the absolute upper limit (CU1354; likelihood ratio: 0). Figure no. 3 shows the probability distribution derived, Figure no. 4 the cumulative probability distribution.

Assuming that market participants are expected to select a confidence level of 80% for the first period (percentile 20, VaR80), for the second a VaR85 (percentile 15), the third a VaR86 (percentile 14) and the fourth a VaR87 (percentile 13), then the net present value is reached by discounting the cash flow of CU613.9 (first year), CU575.7 (second year), CU566.1 (third year) and CU556.6 (fourth year) with the (constant) risk free interest rate of 0.05. The net present value is CU2,054 (when disregarding capital outlay).

As digression: the net present value following Method 2 is reached by discounting the expected value with a risk-adjusted interest rate. The net present value is equal to CU2,583 under the assumptions of a constant risk-adjusted interest rate of 0.08 and a constant probability distribution over the four periods (when disregarding capital outlay).

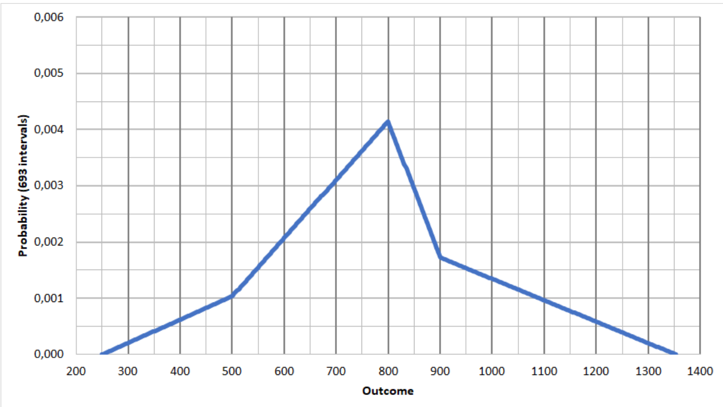

Figure no. 3 – Probability distribution (technique 2)

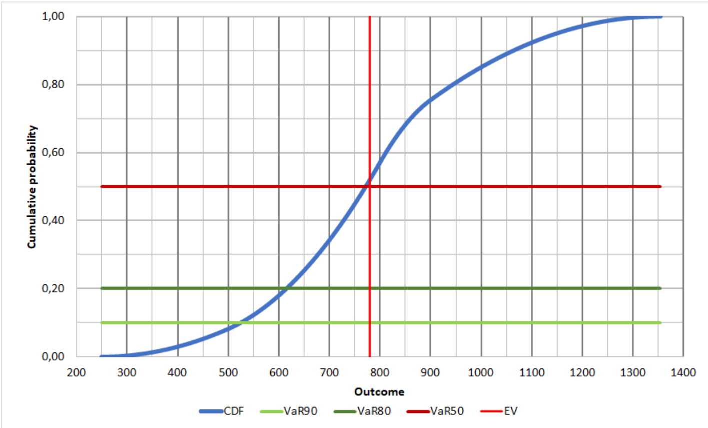

Figure no. 4 – Cumulative probability distribution (technique 2)

4. AN ATTEMPT TO DERIVE THE VALUES AT RISK FOR THE DETERMINED HISTORICAL CERTAINTY EQUIVALENT COEFFICIENTS

In order to interpret the derived certainty equivalent coefficients not only as historically observed safety margins on the expected value but also as indications of a Value at Risk, a simulation study is conducted. The study follows the rational outlined in Section 1. Under the premise that a certainty equivalent is represented by a Value at Risk with a certain confidence level, the confidence level of the value at risk for a certainty equivalent can conversely be determined from a probability distribution. In the derivation of the Values at Risk for the calculated historical certainty equivalence coefficients, it is assumed that the yields on corporate bonds are based on an investor's assessment of the company's possible income and expenses (IFRS Foundation, 2022b).

The modelling is based on the following framework conditions: (a) The median of the probability distribution has to be either identical with the expected value (a symmetrical distribution) or has to be left of the expected value (a right skewed probability distribution). The latter is because, in the case regarded, higher numerical values represent more favourable values. In a left skewed probability distribution, the median lies to the right of the expected value. This would mean the Value at Risk with the lowest confidence level (VaR50) would indicate a more favourable outcome than the expected value. From this would follow a negative cash risk premium, a situation that is incompatible with Method 1, in which the risk-adjusted expected cash flows always have a lower value than the expected cash flows (IFRS13.B25). Therefore, a right skewed probability distribution is chosen, meaning that the expected value is situated right to the median and is representing a Value at Risk with a confidence level of less (and not higher) than 50%.

(b) The probability distributions must be calibrated to the left of the expected value, as this side is decisive for deriving a Value at Risk in the case regarded (higher numerical values represent more favourable outcomes). For calibration, the recommendation of the European Central Bank from May 2015 is taken to put a valuation haircut on company bonds of 30% from the market value if these are accepted as security (European Central Bank, 2015; European Union, 2024). That safety margin is interpreted as (the Bank's) certainty equivalent coefficient of the company's possible income and expenses, representing a VaR99 (percentile1) or a VaR95 (percentile5), following Jorion (2007, p. 119).

Based on these considerations, four distributions are modelled: (a) normal distribution with the valuation haircut (CEC= 0.7) at percentile 1 (Study 1), (b) normal distribution with the valuation haircut (CEC= 0.7) at percentile 5 (Study 2), (c) lognormal distribution with the valuation haircut (CEC= 0.7) at percentile 1 (Study 3), (d) lognormal distribution with the valuation haircut (CEC= 0.7) at percentile 5 (Study 4).

The normal distribution represents a situation that the market participants consider both a favourable and an unfavourable deviation to be equally likely. The selection of the lognormal distribution follows studies in which the variability of cost variations in major acquisition projects were modelled (Garvey *et al.*, 2012; Lee *et al.*, 2012; Kasprik, 2024). It represents a situation that the market participants consider a variability on the favourable side (higher values) as more likely than on the unfavourable side.

Regarded is a bandwidth between 0-times the expected value and 2-times the expected value with the expected value reaching the numerical value of one, so that the x-values left to the expected value represent the certainty equivalent.

The normal distribution is derived from a (0;1)-standard normal distribution, discretised in 969 equal width intervals, by varying the z-value to reach the desired percentile at the 340th interval representing the value 0.7-times the expected value when transforming the z-values between zero and two. The functional values are normalized to represent a probability distribution.

The lognormal distribution is derived from iteratively adjusting the scale parameter alpha and the shape parameter sigma after discretising the bandwidth in 1000 equal width intervals so that the input value of one is equal to the expected value as well as the cumulative probability of the input value of 0.7 is either 0.01 (Study 3) or 0.05 (Study 4). The functional values are normalized to represent a probability distribution.

The derivation of the probability distributions is done by support of Microsoft Excel Professional Plus 2021.

Annex 2 gives the distribution parameters as well as selected location parameters of the modelled distributions. The certainty equivalents for VaR50, Var60, VaR70, VaR80, and VaR90 are displayed in Table no. 2. Figure no. 5 and Figure no. 6 graphically show the modelled probability distributions.

Table no. 2 – Certainty equivalent coefficients for different Values at Risk

	Normal (Study 1) Valuation haircut (CEC= 0.7) at P1	Normal (Study 2) Valuation haircut (CEC= 0.7) at P5	Lognormal (Study 3) Valuation haircut (CEC= 0.7) at P1	Lognormal (Study 4) Valuation haircut (CEC= 0.7) at P5
CEC VaR50	1.0000	1.0000	0.9895	0.9795
CEC VaR60	0.9670	0.9546	0.9535	0.9295
CEC VaR70	0.9319	0.9051	0.9155	0.8796
CEC VaR80	0.8927	0.8473	0.8736	0.8256
CEC VaR90	0.8349	0.7668	0.8176	0.7536

5. ANALYSIS AND DISCUSSION

The analysis concentrates on the extreme classes derived in Section 4: firstly, the class with the highest certainty equivalent coefficients observed, which solely contain AAA-rated bonds (class 1), and, secondly, the three lower classes, which solely contain the certainty equivalent coefficients of Baa-rated bonds in recession phases (class 9 to 11). The certainty equivalent coefficients of class 9 to 11 are aggregated, resulting in a weighted CEC-average of 0.9494.

Table no. 3 shows the Values at Risk for these certainty equivalent coefficients derived from the four simulations described in Section 4. Uniformly, the Values at Risk for AAA-rated bonds in class 1 lie above those of Baa-rated bonds in a recession phase. This result is explained that, in contrast to the Baa-rated bonds, in a situation where the financial stability and the market position of a company, and, further, the economic situation are assessed as favourable, a market participant sees the utility of possible outcomes as lying near the expected value (in class 1: 0.9932 times the expected value) and therefore accepts a Value at Risk with a confidence level of about 50%.

However, the derived Values at Risk for the aggregated group of Baa-rated bonds yields in a recession phase (the weighted mean of the classes 9 to 11) are also between VAR56 and VaR65 and appear to be at a rather low level (see Table no. 3), especially when taking into account that the certainty equivalents coefficients with a value of lower than 0.9661 (class 7) represent outliers (see Table no. A1 in Annex 1).

Table no. 3 – Values at Risk of the certainty equivalent coefficients derived for the outer classes of AAA- and Baa-rated corporate bonds

	Normal (Study 1) Valuation haircut (CEC= 0.7) at P1	Normal (Study 2) Valuation haircut (CEC= 0.7) at P5	Lognormal (Study 3) Valuation haircut (CEC= 0.7) at P1	Lognormal (Study 4) Valuation haircut (CEC= 0.7) at P5
Class 1 (CEC: 0.9932)	VaR52	VaR51	VaR49	VaR47
Mean of classes 9 to 11 (weighted) (CEC: 0.9494)	VaR65	VaR61	VaR61	VaR56
First Outlier (left side) (CEC: 0.9661)	VaR60	VaR57	VaR56	VaR52

To get an impression whether the Values at Risk derived for Baa-rated bond yields in a recession phase might be regarded as less representative for corporate bonds rated as riskier investment, a further estimation, then regarding U.S. high-yield bonds, is realised. Chosen is the ICE BofA index (FRED, 2024). The index gives the spreads between a computed option-adjusted spreads index of U.S. bonds that are below investment grade (rated BB or below) and a spot treasury curve on daily basis. As comparison standard, the market yields on U.S. treasury securities at 10-year constant maturity (FRED, 2025c) are chosen.

The option-adjusted spreads indicate the interest rate premium and cannot be regarded as the risk-adjusted interest rate (the sum of the interest rate premium and the risk-free interest rate), necessary for applying formula (6). Therefore, on daily basis, the corresponding market yields on U.S. treasury securities (representing risk-free interest rate) are added to the U.S. high-yield bonds option-adjusted spreads, for getting an indication of the risk-adjusted interest rate. Based on this transformation, the certainty equivalent coefficients are calculated from formula (6).

Regarded are 6,900 data points from 1996-12-31 up to 2024-08-15. The certainty equivalent coefficients gained as well as location parameters of the CEC-frequency distribution are given in Annex 3.

In contrast to the results gained by regarding AAA- and Baa-bond yields, a relationship of the certainty equivalent coefficients to a recession phase (following the definition of FRED (2025a, 2025b)) could not be observed: in the recession 2001-03-01 to 2001-11-01 the averaged CEC is 0.9258, in the recession 2007-11-30 to 2009-06-02 the averaged CEC is 0.9022, and in the recession 2020-01-31 to 2020-04-01 the averaged CEC is 0.9441.

For this reason, the Values at Risk for the first outlier on the left side as well as the second quartile of the CEC-frequency distribution are taken for checking the plausibility. The Values at Risk of the first outlier on the left side, reaches considerably higher values between 64 % (lognormal distribution with valuation haircut at P5) and 76% (normal distribution with valuation haircut at P1) (see Table no. 4) than the Values at Risk derived for Baa-rated bonds in a recession phase (mean of classes 9 to 11: VaR56 to VaR65; see Table no. 3). The Values at Risk of the second quartile (VaR54 to VaR62; see Table no. 4) are in nearly the same range of the Values at Risk derived for Baa-rated bonds in a recession phase.

The results are interpreted that they support the results gained in Study 1 to 4.

Table no. 4 – Values at Risk of the certainty equivalent coefficients derived for CEC-quartiles of U.S. high-yield bonds

	Normal (as in Study 1) Valuation haircut (CEC= 0.7) at P1	Normal (as in Study 2) Valuation haircut (CEC= 0.7) at P5	Lognormal (as in Study 3) Valuation haircut (CEC= 0.7) at P1	Lognormal (as in Study 4) Valuation haircut (CEC= 0.7) at P5
First outlier (left side) (CEC: 0.9077)	VaR76	VaR69	VaR71	VaR64
First quartile (P25) (CEC: 0.9426)	VaR67	VaR62	VaR62	VaR57
Second quartile (P50) (CEC: 0.9570)	VaR62	VaR59	VaR58	VaR54
Third quartile (P75) (CEC: 0.9658)	VaR60	VaR57	VaR56	VaR52

A second supplementary study is conducted to scrutinise the assumption in Study 1 and Study 3 that the certainty equivalent coefficient of 0.7 stands for a VaR99 and the assumption in Study 2 and Study 4 that the certainty equivalent coefficient of 0.7 stands for a VaR95. Analysed is the case that the investors have had regarded the certainty equivalent coefficient of Moody-Baa-rated bond yields in a recession phase (0.9494, the mean of classes 9 to 11; see Table no. 3) as Value at Risk with a confidence level of 80% - following the recommendation in project budgeting (see Section 1).

Therefore, the parameters of the normal (analysis 1) and lognormal (analysis 2) distribution are modelled so that the distributions' x-value of 0.9494 corresponds to percentile 20 (the VaR80). The parameters are given in Annexes.

The then derived Values at Risk for the certainty equivalent coefficients of Moody-AAA-rated bond yields (0.9932 in class 1; see in Table no. 1) are, as expected, laying above those derived in Study 1 to 4 (see Table no. 3) but still within a plausible range: In analysis 1 (normal distribution) VaR54 is gained, in analysis 2 (lognormal distribution) VaR53.

However, this possible assumption is contradicted by the extreme skewness of the probability distributions derived in analysis 1 as well as in analysis 2 (see Figure no. 5 and Figure no. 6). The probability distributions' coefficients of variation (normal distribution: 0.057, lognormal distribution: 0.058) are fare below analogues studies in which the variation of expected and reached outcomes are analysed. There, coefficients of variation between 0.15 and 0.45 are reported and considered plausible (Thomas and Fitch, 2014). It is to mention that the coefficient of variations in Study 1 to 4 approximately lay within this range (Study 1: 0.1282, Study 2: 0.1813, Study 3: 0.1490, Study 4: 0.2053), indicating a plausible modelling.

Based on these analyses, it is concluded that a confidence level between 60% (based on the CEC of classes 9 to 11 in Study 1 to 4) and 75% (based on the CEC of the first outlier on the left side in the supplementary study; see Table no. 4) can be regarded as the upper level of the market participants' expectations with regard to the non-diversifiable risk to be compensated.

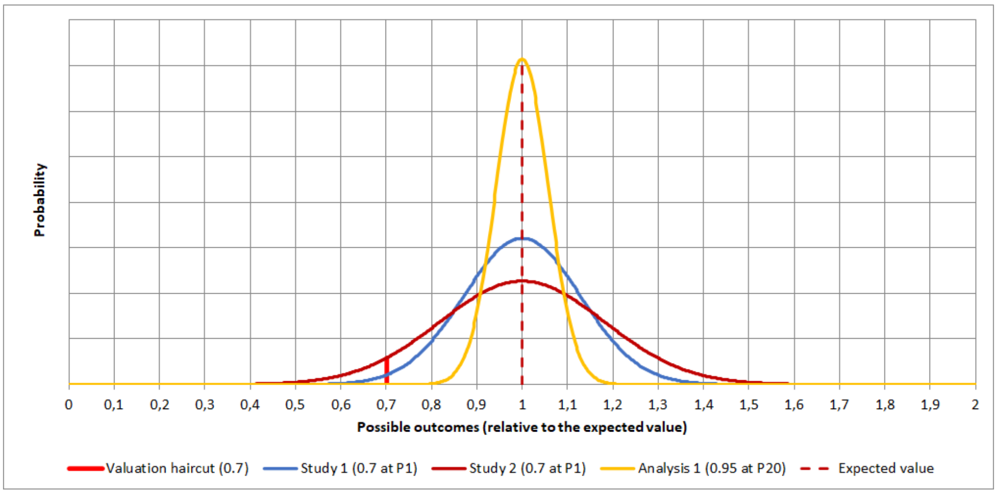

Figure no. 5 – The normal probability distributions modelled

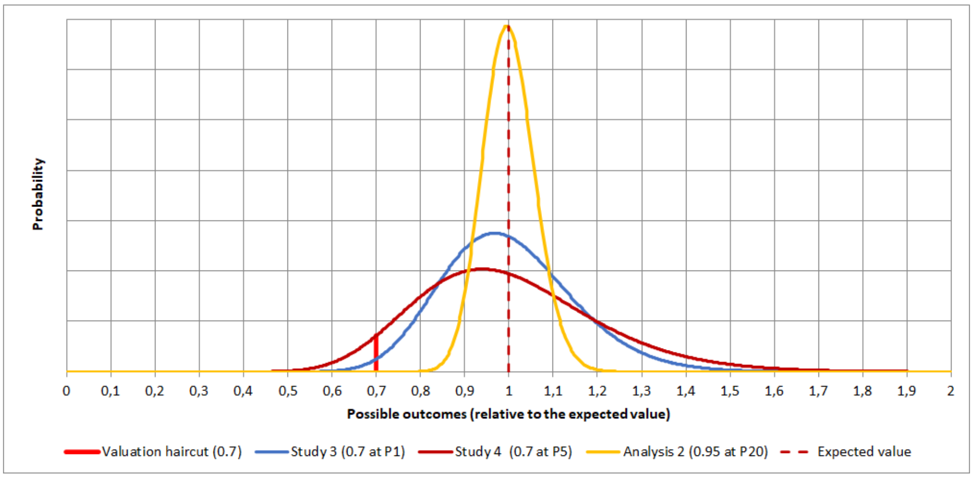

Figure no. 6 – The lognormal probability distributions modelled

It is to mention that if Values at Risk with confidence levels of above 80% might be requested or accepted then, following this analysis, relative safety margins on the expected value from 11 to 17% (VaR80; see Table no. 2) and from 17% to 25 % (VaR90; see Table no. 2) would be the consequence. In the author's view, this deviates drastically from the relative safety margin of 5% derived for Baa-rated bonds in a recessionary phase, which already are far outside the interquartile range: more than three times the interquartile range below the first quartile.

6. CONCLUDING DISCUSSION

The results suggest that a certainty equivalent which represents a confidence level between 60 and 75% should be accepted and/ or requested when deriving a fair value applying Method 1 of expected present value techniques. The certainty equivalents derivable from historic interest rate premiums of corporate bonds yields rated as AAA, Baa or below investment grade can be attributed to Values at Risk with a certainty level of 60 to 75%. This result is based on theoretical considerations, and, therefore, must be interpreted as starting point for further research interrogating the modelled probability distributions.

If one follows the opinion that the modelled probability distributions give a realistic view, then one could nevertheless take the view that the confidence levels derived in the studies are unrealistically low (in comparison to the expectations formulated in risk management practice, see Section 1).

A first explanation could be that market participants had diversified their investment, had regarded the systematic (i.e., market) risk as low and had, therefore, accepted a low confidence level.

A second explanation could be: Market participants have had derived their certainty equivalent not solely on regarding possible future income and expenses (resp. cash flows), but also on other benefits which were not incorporated in the monetary quantified possible outcomes (following Fioretti (2012)). Should this be the case, however, the income approach itself would be questionable: formally, market participants include all benefits in the possible but uncertain cash flows resp. income and expenses (IFRS Foundation, 2022b).

A third explanation could be that the observed low confidence levels would have been caused by the risk-measure Value at Risk itself. The risk-measure Value at Risk evaluates possible but uncertain outcomes by solely regarding the most unfavourable outcome of the group of outcomes which is defined by their cumulative probability of occurrence (determined by the selected confidence level), if all outcomes are considered in ascending order of advantage. No utility is ascribed to the outcomes which are more favourable and which are less favourable than the Value at Risk.

This explanation also cannot be ruled out. However, if one follows this explanation, then one has to accept a lower degree of transparency if an entity applies Method 1: neither a granular cumulative probability distribution nor the confidence level must be disclosed (see Section 3).

Based on this discussion, the author sees the need for further empirical studies analysing market expectations when investors invest in bonds of different risk classes. A frequency distribution of the belief as to whether the investor's plan will underperform, outperform or exceed the plan could help to verify the estimated probabilities for both Method 1 and Method 2. In addition, this could lead to a revision of the assumption used in this study that a confidence level of 50 % (the median) must be equal to or lower than the expected value (see Section 4).

As a side note, the author interprets the results as an indication of the question raised in the literature of how to quantify a decision that is 'more likely than not' (Bohušová *et al.*, 2014) or 'about as likely as not' (Mastrandrea *et al.*, 2010) to lead to the planned outcome or that is 'reasonably certain' to lead to the planned outcome (IFRS Foundation, 2024). He suggests that a distinction in relation to the risk measure Value at Risk is not possible if the decision is based on possible future cash flows or income and expenses. Values at risk with a confidence level of 60 to 75% describe the prevailing degree of certainty from a market perspective.

7. SUMMARY

The study raised the point to elicit thresholds for certainty equivalents when assessing the fair value with Method 1 of the present value techniques within the methodology of income approaches (the certainty equivalent method). Through applying the risk-measure Value at Risk as indicator for certainty equivalents, it is tried to utilise experience from risk management practice.

In order to come to an assessment of the expectations of market participants, the certainty equivalents of investors in corporate bonds were calculated from Moody-AAA- and of Moody-Baa-rated U.S. corporate bond yields to U.S. treasury securities at 10-year constant maturity. For scrutinising the results, an own estimation of bond yields of ICE-rated U.S. high-yield bonds to the market yield on U.S. treasury securities at 10-year constant maturity was realised.

The results show that in case of the Moody-AAA- and of Moody-Baa-rated U.S. corporate bond yields in the time span analysed (1996-2024) the lowest certainty equivalent coefficients were observed for Baa-rated bonds in a recession phase: 0.9459 (weighted average). This corresponds to a rounded relative safety margin of 5% on the expected value. In case of the ICE-rated U.S. high-yield bonds regarded (1997-2024) the certainty equivalent coefficient of the second quartile reaches a similar magnitude: 0.9570, a remarkable difference is observable at the first outlier (minus 1.5-times the interquartile change from the first quartile): CEC: 0.9077 (a relative safety margin of 9% on the expected value).

The author attempted to interpret the observed certainty equivalent coefficients as Value at Risk with a confidence level corresponding to the certainty equivalent coefficients observed. Four studies und two supplementary analyses led to the result that a confidence level between 60% (Baa-rated bonds in a recession phase between 1996-2024) and 75% (first outlier-observation in ICE-rated U.S. high-yield bonds between 1997-2024) can be regarded as the upper level for the expectations of market participants with regard to the non-diversifiable risk to be compensated.

The proposition in risk management practice, to utilize a confidence level of 80% or higher when evaluating possible but uncertain outcomes, is not supported by this study and cannot be assessed as the market participants' confidence level.

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ANNEX 1
MOODY-AAA- AND MOODY-BAA-RATED BOND YIELDS

Table no. A1 – Location parameters of the CEC-frequency distribution

	CEC
Percentile1	0.9641
First outlier (-1.5 times the IQR from P25)	0.9661
First quartile (P25)	0.9792
Median (P50)	0.9837
Third quartile (P75)	0.9879
Percentile99	0.9940
“Outlier” (Percentile100; +1.068 times the IQR from P75)	0.9972
Average	0.9830

IQR: Interquartile range

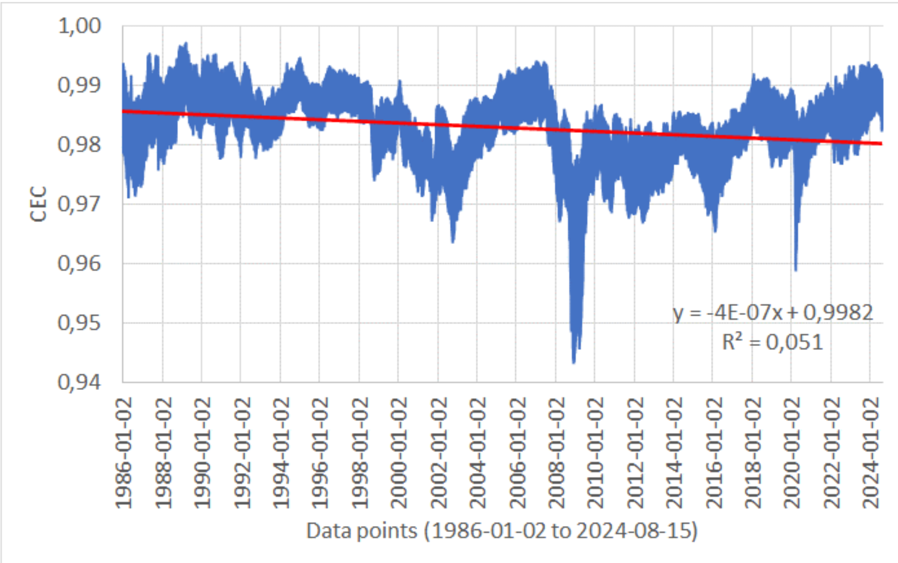

Figure no. A1-1 – Certainty equivalent coefficients of AAA- (upper limit) and Baa-rated (lower limit) U.S. corporate bond yields from 1986-01-02 to 2024-08-15

Table no. A2 – Distribution parameters of the normal probability distributions

	Study 1: Valuation haircut (CEC= 0.7) at percentile1	**Study 2: Valuation haircut (CEC= 0.7) at percentile5**
z-value (+ / -)	7.8001	5.5151
Mode	1.0000	1.0000
Median (P50)	1.0000	1.0000
Expected value	1.0000	1.0000
Coefficient of Variation	0.1282	0.1813
Yules Coefficient of Skewness	0.0000	0.0000

Table no. A3 – Location parameters of the normal probability distributions

	Study 1: Valuation haircut (CEC= 0.7) at percentile1	Study 2: Valuation haircut (CEC= 0.7) at percentile5
Percentile1	0.7007	0.5789
First quartile (P25)	0.9133	0.8782
Median (P50)	1.0000	1.0000
Third quartile (P75)	1.0867	1.1218
Percentile99	1.2972	1.4211

Table no. A4 – Distribution parameters of the lognormal probability distributions

	Study 3: Valuation haircut (CEC= 0.7) at percentile1	Study 4: Valuation haircut (CEC= 0.7) at percentile5
Scale parameter alpha	0.9891	0.9797
Shape parameter sigma	1.1597	1.2261
Mode	0.9675	0.9395
Median (P50)	0.9895	0.9795
Expected value	1.0000	1.0000
Coefficient of Variation	0.1490	0.2053
Yules Coefficient of Skewness	0.0505	0.0667

Table no. A5 – Location parameters of the lognormal probability distributions

	Study 3: Valuation haircut (CEC= 0.7) at percentile1	Study 3: Valuation haircut (CEC= 0.7) at percentile5
Percentile1	0.7017	0.6097
First quartile (P25)	0.8956	0.8536
Median (P50)	0.9895	0.9795
Third quartile (P75)	1.0935	1.1234
Percentile99	1.3953	1.5712

ANNEX 2
U.S. HIGH-YIELD-RATED BONDS

Table no. A6 – Certainty equivalent coefficients for yields of U.S. high-yield bonds to the market yield on U.S. treasury securities at 10-year constant maturity (1986-01-02 to 2024-08-15)

	CEC (group average)	Relative frequency
Class 1	0.9691	0.3267
Class 2	0.9574	0.3236
Class 3	0.9433	0.1722
Class 4	0.9297	0.1113
Class 5	0.9163	0.0357
Class 6	0.9044	0.0080
Class 7	0.8887	0.0029
Class 8	0.8710	0.0029
Class 9	0.8606	0.0107
Class 10	0.8461	0.0033
Class 11	0.8327	0.0028

Table no. A7 – Location parameters of the CEC-frequency distribution

	CEC
Percentile1	0.8596
First outlier (-1.5 times the IQR from P25)	0.9077
First quartile (P25)	0.9426
Median (P50)	0.9570
Third quartile (P75)	0.9658
Percentile99	0.9762
"Outlier" (Percentile100; +0.5036 times the IQR from P75)	0.9776
Average	0.9516

IQR: Interquartile range

ANNEX 3
COUNTER-CHECK OF THE STUDY'S PROBABILITY DISTRIBUTIONS

Table no. A8 – Distribution parameters of the probability distributions under the condition that the CEC of Baa-rated bonds (0.9494) corresponds with VaR80

	Analysis 1: Normal distribution	Analysis 2: Lognormal distribution
Scale parameter alpha		0.9983
Shape parameter z resp. sigma	17.3517	1.0600
Mode	1.0000	0.9955
Median (P50)	1.0000	0.9975
Expected value	1.0000	1.0000
Coefficient of Variation	0.0576	0.0583
Yules Coefficient of Skewness	0.0000	0.0256

Table no. A9 – Location parameters of the probability distributions under the condition that the CEC of Baa-rated bonds (0.9494) corresponds with VaR80

	Analysis 1: Normal distribution	Analysis 2: Lognormal distribution
Percentile1	0.8658	0.8716
First quartile (P25)	0.9608	0.9595
Median (P50)	1.0000	0.9975
Third quartile (P75)	1.0392	1.0375
Percentile99	1.1342	1.1434