Scientific Annals of Economics and Business
70 (2), 2023, 221-234
DOI: 10.47743/saeb-2023-0022

The Effectiveness of the Huber's Weight on Dispersion and Tuning Constant: A Simulation Study

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Abstract: Dispersion measurement and tuning constants are critical aspects of a model's robustness and efficiency. However, in the presence of outliers, the standard deviation is not a reliable measure of dispersion in Huber's weight. This research aimed to assess the efficacy of the Huber weight function in terms of dispersion measurement and tuning constant. The simulation study was conducted on a hybrid of the autoregressive (AR) model and the generalized autoregressive conditional heteroscedasticity (GARCH) model with 10% and 20% additive outlier contamination. In the simulation analysis, three dispersion measurements were compared: median absolute deviation (MAD), interquartile range (IQR), and IQR/3, with two tuning constant values (1.345 and 1.5). The numerical simulation results showed that during contamination with 10% and 20% additive outliers, the IQR/3 outperformed the MAD and IQR. Our findings also showed that IQR/3 is a potentially more robust dispersion measurement in Huber's weight. The tuning constant of 1.5 revealed a decrease in resistance to outliers and increased efficiency. The proposed IQR/3 model with a constant tuning value (h) of 1.5 outperformed the AR(1)-GARCH(1,2) model while minimising the effect of additive outliers.

Keywords: dispersion; tuning constant; Huber; generalized autoregressive conditional heteroscedasticity; additive outliers.

JEL classification: C15; C52; C53.

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Article history: Received 16 May 2022 | Accepted 28 May 2023 | Published online 19 June 2023

To cite this article: Ghani, I.M.M., Rahim, H.A. (2023). The Effectiveness of the Huber's Weight on Dispersion and Tuning Constant: A Simulation Study. *Scientific Annals of Economics and Business*, 70(2), 221-234. https://doi.org/10.47743/saeb-2023-0022.

1. INTRODUCTION

M-estimator is a common approach used in the robust method. It has been discovered that M-estimators are more computationally efficient, as described in the Barrow *et al.* (2020) study. The majority of researchers used M-estimator in a variety of fields, including finance and econometrics (Fan *et al.*, 2019), geodesy and surveying (Osada *et al.*, 2018), business survey (Dehnel, 2016), hydroelectric power (Erdoğan, 2012), mechanical systems (Pennacchi, 2008), infrared spectroscopic application (Pell, 2000) and biological experimentation (Elsaied & Fried, 2016). Some researchers verified M-estimator via simulation experiment to improve their study (Erdoğan, 2012; Elsaied & Fried, 2016; Ghazali *et al.*, 2017; Ertaş, 2018; Polat, 2020). There are several robust methods, such as least absolute deviation (LAD) (Edgeworth, 1887), M-estimator (Huber, 1964), R-estimator (Jaeckel, 1972), least median of squares (LMS) (Rousseeuw, 1984), least trimmed squares (LTS) (Rousseeuw, 1984), S-estimator (Rousseeuw & Yohai, 1984), and MM-estimator (Yohai, 1987). Since they could provide some protection against outliers, both LAD-estimator and M-estimator are popular alternatives in the context of robust estimation of time series models (Barrow *et al.*, 2020). However, M-estimator is preferred because it is simple and straightforward (Osada *et al.*, 2018). Thus, many remarkable results had reported the M-estimator's potential, mainly Huber's function.

Huber's M-estimator has three functions: objective, influence, and weight. The weight function is a fundamental component of a particular implementation of the M-estimation, reweighting observations affected by outliers throughout the iteration process (Osada *et al.*, 2018). The Huber's weight function has been widely investigated (Pell, 2000; Dehnel, 2016; Ghazali *et al.*, 2017; Osada *et al.*, 2018; Polat, 2020; Wada, 2020). Huber's weight function residual was typically standardised using mean and standard deviation as the central tendency and dispersion, respectively. In the presence of outliers, however, both measures are non-robust (Hedayat & Su, 2012), have overestimated values (Dehnel, 2016), and are extremely sensitive to outliers (Park & Leeds, 2016). Furthermore, Hedayat and Su (2012) found that a wide range of tuning constant and dispersion measure options makes it challenging to try and convince people, particularly non-statisticians. Hence, there may be another robust dispersion measurement and tuning constant that can be considered to examine the effectiveness of the econometric model in the presence of outliers.

Moreover, the model's effectiveness is related to its robustness and efficiency. The MAD is a measurement of robust dispersion. According to Rousseeuw and Croux (1993), the median and MAD are simple and easy to compute but extremely useful. Aside from MAD, the IQR was proposed as a robust dispersion in the simulation study by Park and Cho (2003). However, the literature review reveals that Huber's weight is dependent on the MAD as a dispersion measurement with the default tuning constant. Therefore, this research aimed to examine the efficiency of Huber weights while taking dispersion measurement and tuning constant into account. The AR and GARCH models were used to validate the robust dispersion measurement in the Huber weight function. Consequently, IQR/3 was proposed as an alternative to the robust dispersion measures (MAD and IQR). Finally, the efficiency of the Huber weight was then compared in terms of the proposed tuning constant value (1.5) versus the default tuning constant value (1.345).

The sections that follow are divided into different sections. The AR (c), GARCH (m,n) model, M-estimator, tuning constant, and performance measurement are all briefly defined in

Section 2. Section 3 described the process in the simulation study in detail. Section 4 contains the results and discussion based on the simulation analysis, while Section 5 concludes the findings.

2. MATERIALS AND METHODS

This section addresses the hybrid of econometric models: AR and GARCH. The AR(c) can be expressed using notation as presented by Box *et al.* (2016):

$$Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + \cdots + \alpha_c Y_{t-c} + \varepsilon_t \quad (1)$$

with α_0 represents a coefficient term, α_c is the AR component coefficient of order c, and ε_t is the white noise at time t. The c order is non-negative integers.

Let P_t represent a random sample of size, T, with $t = 1, \dots, T$. Considering the GARCH (m, n) model developed by Bollerslev (1986), the equations can be written as follows:

$$r_t = Y_t + \varepsilon_t \quad (2)$$

$$\varepsilon_t = \sigma_t X_t,\ X_t \sim N(0,1) \quad (3)$$

$$\sigma_t^2 = \phi_0 + \phi_1 \sigma_{t-1}^2 + \cdots + \phi_m \sigma_{t-m}^2 + \varphi_1 \varepsilon_{t-1}^2 + \cdots + \varphi_n \varepsilon_{t-n}^2 \quad (4)$$

with r_t is the return series ($\ln(P_t/P_{t-1})$), Y_t is a conditional mean, ε_t is a residual term at time t, X_t is the residual standardised, σ_t^2 is the conditional variance at time t, ϕ_0 is the constant coefficient, σ_{t-1}^2 is the previous variance currently predicted, and ε_{t-1}^2 is the new details on volatility observed at the earlier moment under these conditions: $\phi_0 > 0, \phi_1, \dots, \phi_m \geq 0$, and $\varphi_1, \dots, \varphi_n \geq 0$.

The additive outliers (AO) affect a single observation since only the T^{th} observation period is affected (Chang *et al.*, 1988; Chan, 1992; Chen & Liu, 1993a; Balke & Fomby, 1994; Caroni & Karioti, 2004; Charles, 2008; Hotta & Tsay, 2012; Kamranfar *et al.*, 2017; Urooj & Asghar, 2017). These outliers could be due to an error in documentation caused by other external factors such as human error or machine malfunction (Lee & Van Hui, 1993; Franses & Van Dijk, 2000; Basu & Meckesheimer, 2007; Urooj & Asghar, 2017). Additionally, the AO specifies an extraneous/exogenous corrective (Urooj & Asghar, 2017) and a gross error model (Hillmer, 1984; Chang *et al.*, 1988).

Through Eq. (4), the model GARCH (m, n) can be written as an AR moving average for ε_t^2 as described by Bollerslev (1986):

$$\varepsilon_t^2 = \phi_0 + \sum_{p=1}^{c} (\phi_p + \varphi_p) \varepsilon_{t-p}^2 + \gamma_t - \sum_{q=1}^{n} \phi_q \gamma_{t-q}^2 \quad (5)$$

with $c = max\,\{m, n\}$ and $\gamma_t = \varepsilon_t^2 - \sigma_t^2; t = 1,2, \dots, n$, where ε_t^2 is the outlier free time-series while γ_t is the outlier-free residuals. Next, Eq. (5) can be designed as:

$$\varepsilon_t^2 = \frac{\phi_0}{1 - \varphi(D) - \phi(D)} + \frac{1 - \phi(D)}{1 - \varphi(D) - \phi(D)} \gamma_t = \frac{\eta}{1 - \varphi(D) - \phi(D)} + \eta^{-1}(D) \gamma_t \quad (6)$$

with $\varphi(D) = \sum_{q=1}^{n} \varphi_q D^q, \phi(D) = \sum_{p=1}^{m} \phi_p D^p$ and $\eta(D) = \frac{1 - \varphi_q(D) - \phi_p(D)}{1 - \phi_p(D)}$.

Once AO is apparent in the part of the GARCH model, the Equation could be transformed into Eq. (7), as shown by Chen and Liu (1993b).

$$e_t^2 = \omega_{\mathrm{IO}}\xi_{\mathrm{AO}}(D)I_t(T) + \varepsilon_t^2 \tag{7}$$

From Eq. (7), it is possible to view this as a regression model for ε_t^2 and to reform as:

$$e_t^2 = \omega_{\mathrm{AO}}x_t + \varepsilon_t^2 \tag{8}$$

where:

e_t^2 is an observed series ε_t^2,

ω_{AO} is the magnitude effect of AO, which is $\omega_{\mathrm{AO}}(T) = \chi_T$,

$\xi_{\mathrm{AO}}(D)$ is the dynamic pattern of AO effect, which is $\xi_{\mathrm{AO}}(D) = 1$,

$I_t(T)$ is the predictor function that can clarify AO's impact as $I_t(T) = \begin{cases} 1 & ,t = T \\ 0 & \text{, otherwise} \end{cases}$ (T is the point where AO occurred).

2.1 M-estimator

Huber (1964) developed the M-estimator, a generalisation of the maximum likelihood estimator, as an alternative to minimising the objective function.

$$\min \sum_{i=1}^{n} \rho(\varpi_i) \tag{9}$$

From Eq. (9), ϖ_i is the i-th residual and ρ is a symmetric function with a specific minimum value of zero. It is important to note that the residuals must be standardised. Let:

$$\varpi_i = \frac{x_i - f(\mu)}{g(\sigma)} \tag{10}$$

where $f(\mu)$ is any central tendency measurement function, such as mean, median, or mode. Meanwhile, $g(\sigma)$ is any dispersion measurement function, such as standard deviation, variance, MAD, range, or IQR. The following equations must be solved to simplify Eq. (9),

$$\sum_{i=1}^{n} \psi\left(\frac{x_i - f(\mu)}{g(\sigma)}\right) = 0 \tag{11}$$

$$\sum_{i=1}^{n} \psi(\varpi_i) = 0 \tag{12}$$

whereby $\psi(\varpi_i)$ is the influence function derived from the objective function's first derivative concerning residuals. $\psi(\varpi_i)$ can be calculated using Eq. (13).

$$\psi(\varpi_i) = \frac{\partial \rho(\varpi_i)}{\partial \varpi_i} \tag{13}$$

Next, the weight function is defined as:

$$w(\varpi_i) = \frac{\psi(\varpi_i)}{\varpi_i} \tag{14}$$

where $w(\varpi_i)$ is the derivative of the $\psi(\varpi_i)$.

Now, we decided to minimise the Huber objective function, hence:

$$\rho(\varpi_{\mathrm{H}}) = \begin{cases} \dfrac{\varpi_{\mathrm{H}}^2}{2} & \text{, for } |\varpi_{\mathrm{H}}| \le h \\ h|\varpi_{\mathrm{H}}| - \dfrac{(h)^2}{2} & \text{, for } |\varpi_{\mathrm{H}}| > h \end{cases} \tag{15}$$

With the first derivative, the $\rho(\varpi_H)$ becomes $\psi(\varpi_{\mathrm{H}})$,

$$\psi(\varpi_{\mathrm{H}}) = \begin{cases} \varpi_{\mathrm{H}} & \text{, for } |\varpi_{\mathrm{H}}| \le h \\ h\big(sgn(\varpi_{\mathrm{H}})\big) & \text{, for } |\varpi_{\mathrm{H}}| > h \end{cases} \tag{16}$$

From Eq. (16), the weight function can be expressed as:

$$w(\varpi_{\mathrm{H}}) = \begin{cases} 1 & \text{, for } |\varpi_{\mathrm{H}}| \le h \\ \dfrac{h}{|\varpi_{\mathrm{H}}|} & \text{, for } |\varpi_{\mathrm{H}}| > h \end{cases} \tag{17}$$

where h is a tuning constant with a value of 1.345 that produce 95% efficiency for normally distributed, ε_t.

2.2 The Robust Huber's Weight of Dispersion

Since the outliers could affect the mean and standard deviation as the central tendency and dispersion, respectively, therefore, Hampel (1974) proposed the MAD as a more robust estimate than the sample standard deviation. Chen and Liu (1993b) also examined the MAD approach to achieve a better estimate and considered it an appropriate option (Simpson & Montgomery, 1998) and more robust dispersion (Leys *et al.*, 2013; Ruppert & Matteson, 2015).

Therefore, from Eq. (17), the robust Huber's weight can be expressed in two ways: $w(\varpi_{\mathrm{MAD}})$ and $w\big(\varpi_{\mathrm{IQR}}\big)$.

$$w(\varpi_{\mathrm{MAD}}) = \begin{cases} 1 & \text{, for } |\varpi_{\mathrm{MAD}}| \le h \\ \dfrac{h}{|\varpi_{\mathrm{MAD}}|} & \text{, for } |\varpi_{\mathrm{MAD}}| > h \end{cases} \tag{18}$$

with:

$$\varpi_{\text{MAD}} = \frac{x_i - \text{median}}{\frac{\text{MAD}}{0.6745}} \tag{19}$$

and:

$$w(\varpi_{\text{IQR}}) = \begin{cases} 1 & , \text{ for } |\varpi_{\text{IQR}}| \leq h \\ \dfrac{h}{|\varpi_{\text{IQR}}|} & , \text{ for } |\varpi_{\text{IQR}}| > h \end{cases} \tag{20}$$

with:

$$\varpi_{\text{IQR}} = \frac{x_i - \text{median}}{\text{IQR}} \tag{21}$$

The h value for the robust Huber's weight in Eq. (18) and Eq. (20) is 1.345, resulting in a 95% efficiency for normally distributed, ε_t.

2.3 The Proposed Huber's Weight of Dispersion

Carnero *et al.* (2012) reported that even if the actual procedure for dealing with outliers comes after the estimate phase, outliers can be detected and corrected before the GARCH parameters are estimated. Subsequently, Eq. (10) is modified to:

$$\varpi_{\text{I}} = \frac{x_i - \text{median}}{\text{IQR}/3} \tag{22}$$

where ϖ_{I} is the contamination data.

Therefore, the proposed Huber's weight can be written as:

$$w(\varpi_{\text{I}}) = \begin{cases} 1 & , \text{ for } |\varpi_{\text{I}}| \leq h \\ \dfrac{h}{|\varpi_{\text{I}}|} & , \text{ for } |\varpi_{\text{I}}| > h \end{cases} \tag{23}$$

where proposed Huber's weight in Eq. (23) has a h value of 1.5, resulting in 99.99% efficiency for normally distributed, ε_t.

2.4 Review of h

The weighting function in M-estimator has h that affects its robustness and efficiency (Holland & Welsch, 1977; Huber, 1981; Wang *et al.*, 2007; Elsaied & Fried, 2016; Gajowniczek & Zabkowski, 2017; Li *et al.*, 2021). A higher h-value improves efficiency but reduces robustness to outliers, whereas a lower h-value reduces efficiency but increases robustness to outliers (Elsaied & Fried, 2016; Li *et al.*, 2021). Cummins and Andrews (1995) also observed that a higher value of h reduces robustness to outliers, which is undesirable. However, it reduces the risk of penalising 'good' data, which is helpful. While decreasing the value of h increases robustness to outliers, it also increases the risk of underweighting 'good' data, resulting in information loss.

Generally, the default value of h for Huber weight is 1.345, which produce 95% efficiency for normally distributed. In their study, Cummins and Andrews (1995) reported that the default values of h provided the best efficiency. Holland and Welsch (1977) and Simpson and Montgomery (1998) attempted to calculate the h value in their studies. However, Huber (1981) suggested that the best Huber weight value was at $h = 1.5$, which is between 1 and 2. Therefore, some researchers had adjusted or modified the h value to improve the performance of a specific weight function (Mbamalu *et al.*, 1995). The various of Huber's h values are summarized in Table no. 1.

Table no. 1 – Adjusted h value of Huber weight.

Tuning constant value, h	Sources
1.2	Cantoni and Ronchetti (2001)
1.2107	Pennacchi (2008)
1.25	Street *et al.* (1988); Chi (1994)
1.5	Wang *et al.* (2007); Gajowniczek and Zabkowski (2017)

2.5 Measure of Performance

The performance of the dispersion measurement in Huber weight for AR(1)-GARCH(1,2) model were compared using two goodness-of-fit measures, such as mean square error (MSE) and root mean square error (RMSE).

$$\text{MSE} = \frac{1}{T}\sum_{t=T_1}^{T}(\sigma_t^2 - \hat{\sigma}_t^2)^2 \qquad \text{RMSE} = \sqrt{\frac{1}{T}\sum_{t=T_1}^{T}(\sigma_t^2 - \hat{\sigma}_t^2)^2}$$

Based on the MSE and RMSE measures, T is the total number of observations, T_1 is the initial observation in the evaluation period, σ_t^2 is the actual conditional variance at time t, and $\hat{\sigma}_t^2$ is the predicted conditional variance at time t. A smaller MSE and RMSE under contamination are required for accurate dispersion measurement.

3. A SIMULATION STUDY

We designed simulations to examine the performance of the AR(1)-GARCH(1,2) model in the presence of AO by combining Huber weight function with the following factors:

a) Percentage of AO contamination, P_{AO} = {0%, 10%, 20%}
b) Dispersion measurement: (MAD, IQR and IQR/3)
c) Tuning constant, $h = \{1.345, 1.5\}$

One thousand simulation iterations of three-time series length, $T = \{500{,}1000{,}5000\}$ were generated using R Core Team (2020) software version 3.6.3. The AR(1)-GARCH(1,2) model was considered in the simulation study, which was adapted from Ghani and Rahim (2018) study. The motivation of selection percentages of AO contamination (P_{AO} = 10% and 20%) was also investigated by Muler and Yohai (2008) and Wang *et al.* (2007) in their research. Previously, researchers used $T = \{500, 1000, 5000\}$ during their simulation processes (Grané & Veiga, 2010; Carnero *et al.*, 2012). Therefore, the process of simulation studies was conducted as follows:

1. The AR(1)-GARCH(1,2) model using the *fGarch* package (Wuertz *et al.*, 2020) with parameter values were specified as: $\{\alpha_0 = 0.043, \alpha_1 = -0.312, \phi_0 = 0.011, \phi_1 = 0.913, \varphi_1 = 0.224, \varphi_2 = -0.136\}$.
2. Data were randomly simulated at the beginning of $T = 500$ with a mean and standard deviation of 0 and 1, respectively.
3. AO contamination was 10% of the series. The location of AO was identified by calculating the magnitude with 16σ.
4. The Huber's weight was derived using three different dispersion measurements (MAD, IQR, and IQR/3) with $h = 1.345$ based on the absolute 10% of AO contamination.
5. The new data was obtained by applying Huber's weight.
6. Steps (3) to (5) were repeated before increasing AO contamination to 20%.
7. Coefficients of the AR(1)-GARCH(1,2) model for three situations were estimated using the *garchFit* function in Gaussian error distribution.
8. The performance of the AR(1)-GARCH(1,2) model for three cases was evaluated.
9. Steps (1) to (8) were repeated for different time series lengths, $T = 1000, T = 5000$. All-time series lengths were carried out using 1000 replications.
10. Steps (1) to (9) were repeated by changing $h = 1.5$.

4. RESULTS AND DISCUSSIONS

In this section, the performances of the AR(1)-GARCH(1,2) model using different dispersion measurements and h in the Huber weight function will be discussed based on simulation study.

4.1 Simulation Results

This section will discuss how the proposed dispersion measurement outperformed the MAD and IQR presented by Park and Cho (2003) when using AO contamination of 10% and 20% for $T = 500,\ 1000,\ 5000$. Again, in this simulation, two concerns were addressed: dispersion measurement and h.

4.2 Dispersion Measurement

Table no. 2 depicts the performance of non-Huber weight and dispersion measurements of 0%, 10%, and 20% AO contamination based on MSE. The MSE result for $T = 500$ reported a 54.4922 increase compared to 10% AO contamination (26.5450). IQR/3 reported a minimum MSE value of 0.3753 for $T = 500$ during contaminated 10% AO, a drop of 98.6% when compared to the non-weighting value (26.5450). The results were followed by a 96.2% drop in MAD and a 95.1% drop in IQR.

The MSE for the IQR/3 in the 20% contamination of AO reported a minimum value of 0.4306, followed by MAD and IQR, which were 1.2855 and 1.8402, respectively. Compared to the non-weighting measurements, all three-dispersion measurements decreased by 99.2%, 97.6%, and 96.6%, respectively (54.4922). A similar situation occurred when T=1000 and 5000.

Table no. 2 – MSE for non-Huber weight and dispersion measurement

T	P_{AO} (%)	NW	MAD	IQR	IQR/3	ΔMAD	ΔIQR	ΔIQR/3
500	0	0.91910						
	10	26.5450	0.9974	1.3061	0.3753	- 96.2	- 95.1	- 98.6
	20	54.4922	1.2855	1.8402	0.4306	- 97.6	- 96.6	- 99.2
1000	0	0.91910						
	10	23.5924	0.9612	1.2472	0.3346	- 95.9	- 94.7	- 98.6
	20	52.9005	1.3382	1.8595	0.4390	- 97.5	- 96.5	- 99.2
5000	0	1.00160						
	10	25.4275	1.0434	1.3575	0.3839	- 95.9	- 94.7	- 98.5
	20	49.5896	1.4176	2.0162	0.4871	- 97.1	- 95.9	- 99.0

Note: *T* = time series length, P_{AO} = percentage contamination of AO, NW = non-weighting, ΔMAD = percentage change of MAD, ΔIQR = percentage change of IQR, ΔIQR/3 = percentage change of IQR/3.

Table no. 3 presents the performance of non-Huber weight and dispersion measurement with RMSEs of 0% (without contamination), 10%, and 20% AO contamination. The RMSE value for T=500 increased 43.3% during 20% AO contamination to 7.3819 compared to the 10% AO contamination (5.1522). When T=500 and PAO = 10%, the RMSE values for MAD, IQR, and IQR/3 were 0.9987, 1.1429, and 0.6126, respectively. The IQR/3 had the greatest percentage reduction in RMSE at 88.1%, followed by MAD (80.6%) and IQR (77.8%).

As PAO was increased to 20%, IQR/3 reported a minimum of RMSE of 0.6562, a drop of 91.1% when compared to the non-weighting (7.3819). The MAD and IQR, on the other hand, fell by 84.6% and 81.6%, respectively. When the time series length was increased to 1000 and 5000, a similar situation occurred.

Table no. 3 – RMSE for dispersion measurement in Huber weight.

T	P_{AO} (%)	NW	MAD	IQR	IQR/3	ΔMAD	ΔIQR	ΔIQR/3
500	0	0.9587						
	10	5.1522	0.9987	1.1429	0.6126	- 80.6	- 77.8	- 88.1
	20	7.3819	1.1338	1.3566	0.6562	- 84.6	- 81.6	- 91.1
1000	0	0.9587						
	10	4.8572	0.9804	1.1168	0.5785	- 79.8	- 77.0	- 88.1
	20	7.2733	1.1568	1.3636	0.6626	- 84.1	- 81.3	- 90.9
5000	0	1.0008						
	10	5.0426	1.0215	1.1651	0.6196	- 79.7	- 76.9	- 87.7
	20	7.0420	1.1906	1.4199	0.6979	- 83.1	- 79.8	- 90.1

Note: *T* = time series length, P_{AO} = percentage contamination of AO, NW = non-weighting, ΔMAD = percentage change of MAD, ΔIQR = percentage change of IQR, ΔIQR/3 = percentage change of IQR/3.

The MAD results for the MSE and RMSE were more robust than the IQR at the P_{AO} = 10% and 20%. Tables no. 2 and no. 3 show that the percentage decrement for MAD is more significant than that for IQR. It has been reported that the MAD is an excellent choice for measuring dispersion (Rousseeuw & Croux, 1993; Simpson & Montgomery, 1998). This is supported by a study done by Park and Cho (2003), which discovered that MAD could be identified as a robust dispersion measurement under a normal distribution with contaminated data. However, our results showed that the IQR/3 outperformed the MAD and IQR during contamination with 10% and 20% AO.

4.3 Tuning Constant, h

The performance of the three-dispersion measurement based on MSE between $h = 1.345$ and $h = 1.5$ is shown in Table no. 4. At T=500, P_{AO} = 10%, and $h = 1.345$, the IQR/3 showed the lowest MSE value compared to MAD and IQR. The MSE value for IQR/3, MAD, and IQR was 0.3173, 0.8968, and 1.1878, respectively, decreased by 98.8%, 96.6%, and 95.5% compared to the non-weighting value (26.5450) in Table no. 2. The MSE value for the three-dispersion measurement was lower when $h = 1.345$ rather than $h = 1.5$, as per this data. For P_{AO} = 20%, $h = 1.345$, the minimum MSE value was IQR/3, which had the highest percentage decrement of 99.3%, followed by MAD (97.9%) and IQR (97.0%). When the time series length was increased to 1000 and 5000, a similar situation occurred.

Table no. 4 – MSE and percentage change (in parentheses) for dispersion measurement with $h = \{1.345, 1.5\}$

T	P_{AO}	$h = 1.345$			$h = 1.5$		
		MAD	**IQR**	**IQR/3**	**MAD**	**IQR**	**IQR/3**
500	10%	0.8968 (- 96.6)	1.1878 (- 95.5)	0.3173 (- 98.8)	0.9974 (- 96.2)	1.3061 (- 95.1)	0.3753 (- 98.6)
500	20%	1.1212 (- 97.9)	1.6078 (- 97.0)	0.3641 (- 99.3)	1.2855 (- 97.6)	1.8402 (- 96.6)	0.4306 (- 99.2)
1000	10%	0.8570 (- 96.4)	1.1326 (- 95.2)	0.2829 (- 98.8)	0.9612 (- 95.9)	1.2472 (- 94.7)	0.3346 (- 98.6)
1000	20%	1.1751 (- 97.8)	1.6375 (- 96.9)	0.3711 (- 99.3)	1.3382 (- 97.5)	1.8595 (- 96.5)	0.4390 (- 99.2)
5000	10%	0.9352 (- 96.3)	1.2375 (- 95.1)	0.3242 (- 98.7)	1.0434 (- 95.9)	1.3575 (- 94.7)	0.3839 (- 98.5)
5000	20%	1.2467 (- 97.5)	1.7674 (- 96.4)	0.4113 (- 99.2)	1.4176 (- 97.1)	2.0162 (- 95.9)	0.4871 (- 99.0)

Note: T = time series length, P_{AO} = percentage contamination of AO, the values in parentheses represents the percentage change.

Table no. 5 – RMSE and percentage change (in parentheses) for dispersion measurement with $h = \{1.345, 1.5\}$

T	P_{AO}	$h = 1.345$			$h = 1.5$		
		MAD	**IQR**	**IQR/3**	**MAD**	**IQR**	**IQR/3**
500	10%	0.9470 (- 81.6)	1.0899 (- 78.8)	0.5633 (- 89.1)	0.9987 (- 80.6)	1.1429 (- 77.8)	0.6126 (- 88.1)
500	20%	1.0589 (- 85.7)	1.2680 (- 82.8)	0.6034 (- 91.8)	1.1338 (- 84.6)	1.3566 (- 81.6)	0.6562 (- 91.1)
1000	10%	0.9257 (- 80.9)	1.0643 (- 78.1)	0.5319 (- 89.1)	0.9804 (- 79.8)	1.1168 (- 77.0)	0.5785 (- 88.1)
1000	20%	1.0840 (- 85.1)	1.2796 (- 82.4)	0.6092 (- 91.6)	1.1568 (- 84.1)	1.3636 (- 81.3)	0.6626 (- 90.9)
5000	10%	0.9671 (- 80.8)	1.1124 (- 77.9)	0.5694 (- 88.7)	1.0215 (- 79.7)	1.1651 (76.9)	0.6196 (- 87.7)
5000	20%	1.1165 (- 84.1)	1.3294 (- 81.1)	0.6413 (- 90.9)	1.1906 (- 83.1)	1.4199 (- 79.8)	0.6979 (- 90.1)

Note: T = time series length, P_{AO} = percentage contamination of AO, the values in parentheses represents the percentage change.

Table no. 5 shows the RMSE performance of the three-dispersion measurement between $h = 1.345$ and $h = 1.5$. IQR/3 reported a minimum RMSE value of 0.5633 for $T = 500$, P_{AO} = 10%, and $h = 1.345$, a drop of 89.1% compared to the non-weighting (5.1522). This was followed by a drop in MAD and IQR of 81.6% and 78.8%, respectively. At $T = 500$, P_{AO} = 20%, and $h = 1.345$, the MSE for the IQR/3 was 0.6034, followed by the MAD and IQR, which were 1.0589 and 1.2680, respectively. Compared to the non-weighting measurements, all three-dispersion measurements decreased by 91.8%, 85.75%, and 82.8%, respectively (7.3819). A similar situation occurred when $T = 1000$ and 5000.

This study found that as the proposed tuning constant $(h = 1.5)$ increased in comparison to the default $(h = 1.345)$, the MSE (in Table no. 4) and RMSE (in Table no. 5) for MAD, IQR, and IQR/3 also increased. The findings in Tables no. 4 and no. 5 proved that as the constant tuning increased, the amount of resistance to outliers decreased and efficiency increased (Cummins & Andrews, 1995; Elsaied & Fried, 2016). The current finding also supported Cummins and Andrews (1995) study, which found that increasing h is better since it reduces the risk of correcting "good" data. Our findings revealed that the IQR/3 outperformed MAD and IQR during AO contamination (P_{AO} = $10\%, 20\%$) with $h = \{1.345, 1.5\}$.

5. CONCLUSIONS

This study aimed to examine and compare the efficiency of Huber weights by taking dispersion measurement and tuning constant into account. The following conclusions can be drawn from the computed results:

1) The simulation results concluded that the proposed dispersion measurement IQR/3 in the Huber weight function performed better than MAD and IQR during 10% and 20% AO contamination.

2) The findings demonstrated that the IQR/3 becomes more robust as the percentage of AO contamination and time series length increase.

3) $h = 1.5$ outperformed the default value $(h = 1.345)$, resulting in less resistance to outliers and greater efficiency.

The current study makes several contributions. First, we propose a method for detecting outliers before the estimation process. Second, the precision of IQR/3 as a dispersion measurement and the tuning constant $(h = 1.5)$ in the Huber weight function can help to reduce the effect of an additive outlier. Finally, the proposed Huber's dispersion weight is quick and straightforward to apply, making it an appealing tool for academic and/or practitioner communities

Acknowledgements

The authors would like to thank the anonymous reviewers for their insightful comments and recommendations, which significantly improved the quality of this article.

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