

University Behavior under Borrowing Constraints: The Effect of Students' Abilities Distribution

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Abstract

The interaction between a university and potential students is examined under the assumptions of financial constraints and of students' abilities following the pattern of a triangular distribution. Subsequently, a comparative statics analysis in terms of welfare and vectors composed of three components – namely quality, tuition fee and ability threshold – is provided. Results suggest that the mode of the distribution is an intrinsic part of *equilibria*, and that a human capital maximizing university makes additional efforts in terms of pricing and non-pricing strategies in order to alleviate the inconveniences arising due to financial constraints and non-uniformity in the distribution of students' abilities.

Keywords: triangular distribution of abilities; borrowing constraints; education quality; tuition fees.

JEL classification: D42; H42; I21; I22.

1. INTRODUCTION

The blurred line between demand and supply seems to be a natural feature of educational markets. As the demand derives from potential students' preferences, the supply is usually determined by the characteristics of the human capital. An intrinsic feature of the demand side is the one related to students' abilities. [Rothschild and White \(1995\)](#) rightly consider abilities as inputs into the production of human capital. Indeed, as abilities play a key role in determining output characteristics, a special attention should be devoted to their distribution.

The borrowing or financial constraints should be regarded, on the whole, as obstacles, and thus as other forms of inputs' characteristics, which indirectly affect individuals' schooling decisions ([Fernández, 1998](#); [Romero and Del Rey, 2004](#); [Romero, 2005](#)). Moreover, such type of barriers should be understood as deeply entangled in the individuals' preferences as long as their relative magnitude proportionally affect individuals' propensity for schooling. On the

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other hand, the human capital, as an output and supply representative, will depend on the characteristics of the absorbed demand for education.

Overall, these are all reasons why demand and supply in educational economics can be hardly considered as fully independent from each other. The analysis presented here contemplates this perspective.

In the proposed setup, students' abilities are directly incorporated into the supply via the human capital production function, whereas borrowing constraints are indirectly incorporated into it via deductions in students' endowments. In this regard, students will differ from each other due to their particular bundle, *i.e.*, ability-endowment bundle. In fact, each of these bundles embody input and output in the human capital production process. Therefore, the blurry line between demand and supply in such markets is introduced via the complementarity of students' abilities and university's quality in determining human capital. This note investigates on how decisions of a human capital-maximizing university are influenced by variations on the input's characteristics, which are generally conceived by the ability-endowment distribution. More specifically, this note intends to address the following research questions:

- How does university behavior changes when potential students' abilities are not uniformly distributed?
- What is the effect of financial constraints on the university's choices along different skewness configurations of potential students' abilities distribution?
- What are the overall welfare implications of having various degrees of financial constraints and of abilities' distributional skewness?

The variations on abilities are specified by a triangular distribution; whereas borrowing constraints (BC), which directly affect endowment, are introduced via a direct fee burden on the potential students' budget. *Equilibria* and welfare outcomes are also compared with results of earlier studies, which rely on the assumption of uniformity in the abilities' distribution and, which vary on the proposed scale of imperfect competition.

Results indicate that when quadratic costs for the monopolist (*i.e.*, human capital maximizing university) are presumed, the skewness' parameter of the abilities' distribution turns out to be an intrinsic part of subgame perfect *equilibria* informing so that the university will accommodate its choices (*i.e.*, quality, tuition fee, admission standards) in accordance with exogenous characteristics of the surroundings. The university will also attempt to internalize negative externalities deriving from borrowing constraints and skewed abilities. In other words, the presence of non-uniformity (skewness) in ability distributions and financial constraints may lead to higher efforts from the university with the aim of internalizing negative externalities. Ultimately, it is shown that the non-cooperative setup of the game can unfold a plausible cooperative setup in which tuition fees are set to zero and the university is allowed to select its students' pool. Results obtained here, which are drawn out of a setup of triangularly distributed abilities, confirm the findings reported in [Friebel and Maldonado \(2015\)](#) pointing out, once more, that application of admission criteria can be welfare detrimental.

The proposed approach builds upon the contributions of [Romero and Del Rey \(2004\)](#) and [Romero \(2005\)](#). However, the analysis here is focused on a particular case of monopolistic competition, where a human capital maximizing university solely operates in the higher education market. In more realistic terms, the approach may shed light on scenarios in which university system is tightly regulated and centralized. Indeed, one can, for the sake of simplicity, consider the whole system monopolistic as long as most of universities in the country operate according to the same set of rules, and aim the same (or very similar)

objectives. The reliance on monopolistic behavior is also justified in the intrinsic nature of competition in the higher education markets of developed countries. As Charles Clotfelter has pointed out for the case of US, the market for university education is segmented with students who are seeking admission to elite universities, rarely applying simultaneously to less prestigious universities (Clotfelter, 1999, p. 5).

Furthermore, the assumption about human capital maximization, is quite in line with recent trends of university behavior in places, where decisions on issues such as universities' subsidies are based on welfare criteria, and not narrowly on university's ownership criteria. On this understanding, the proposed approach is aligned with David Dill's viewpoint about trends in universities behavior. Dill (2005) points out that what really matters is the behavior of universities in the marketplace, which in fact is converging towards a type of behavior that aims to balance the preferences of a vast spectrum of agents. Likewise, the latter study reminds that distinctions such as "public vs. private", and "non-profit vs. for profit", for the US higher education market, are blurring:

“[...] the federal government offers competitive contracts with public universities, private universities, and for that matter with profit-making institutions to conduct the research and scholarship that is believed to be in the broader public interest” (Dill, 2005, p. 3).

Similar arguments are also raised by Hazelkorn and Gibson (2019), who discuss, in particular for the case of few leading science and education countries, the reconfiguration of higher education towards a common good. Among other things, they make the point of the growing need for application of national higher education strategies (*e.g.*, performance - based funding) in order to mitigate tensions between private and public interests.

Certainly, it is not pretended that the former views, along with the assumption of human capital striving, will elucidate much of economic relations, which take place in other models of higher education provision. It is obvious that major exceptions are the operation of some universities, especially those of private ownership in some particular parts of the world, where government regulation related to many aspects of universities' behavior are still fragile, unbalanced, and sometimes intrinsically driven by the ownership criteria (on this see for example, Levy, 2005; Lutran, 2007; Klemencic *et al.*, 2015). Nevertheless, I believe that the importance of the proposed approach relies on easing the understanding, at least on a theoretical level, of the relations among features embodied in consolidated education systems, in nations where universities have already acquired the capability to self-regulate and have better aligned their missions with benevolent goals such as human capital maximization, satisfaction of a wider spectrum of stakeholders, equity, and the overall prosperity of the nation.

The model presented here is very much in the same spirit with that of Rothschild and White (1995), in which it is assumed that students are input and at the same time output into the human capital production. It is also in line with more recent and sophisticated models on university competition, such as the one proposed by Epple *et al.* (2006), where colleges strive for quality maximization. However, by assuming that university strives for human capital maximization, the concept of quality as put forward in this note is rather a means to an end than an end in itself.

The note is organized as follows. In the next Section 2, I propose the basic model with the central assumptions, players' preferences, and the setup of interactions. Section 3 reports the *equilibria* of the higher education game, comparative statics, welfare implications, and an

extension on tuition-free systems. [Section 4](#) concludes by summarizing the main findings. All proofs are in the [Annexes](#).

2. THE MODEL AND ITS ASSUMPTIONS

2.1 The Higher Education Game

It is assumed the presence of two players in the higher education market, namely, a university and individuals. The university acts as a monopoly aiming to better pursue its interest given the scarcity of resources (*e.g.*, funds, classrooms, labs, professors, etc.). As stated in the Introduction, I avoid the classical distinction between public and private universities by assuming that a modern university fully exerts its rationality through pursuing human capital maximization. Individuals, on the other hand, interact with the university in order to acquire their education in the most beneficiary way, *i.e.*, with the highest quality and the lowest feasible cost. Consequently, the university (individuals) will rationally react and it (they) will adjust its (their) behavior in response to the choices and behavior of individuals (university) considering the payoffs' structure as given. The interaction, thus, is assumed to be entirely driven by incentives.

2.2 Individuals

The basic framework provided in [Romero and Del Rey \(2004\)](#) and [Romero \(2005\)](#) in defining individuals' (*i.e.*, potential students') measure, and their preferences for higher education, is adopted. The basic setup allows to account for the intended particularities, and at the same time, it sets a benchmark for additional comparative statics.

The utility obtained from matriculating in the university for a potential student i , is

$$u_i = w_i + a_i q - f \quad (1)$$

where the product $a_i q$ measures the human capital, which will be embodied in the potential student i (she) from matriculating at the university; w_i is her initial endowment; and f is the tuition fee to be paid by her to the university. For the potential student, university's quality and tuition fee will be beyond her control since she will have no power to directly affect these. Hence, on the individual level, both endowment and ability will largely determine her reaped utility from matriculating in the university.

A non-uniform distribution of individuals' abilities is assumed, whereas the uniformity in the endowment distribution is assumed to hold across all potential students. This axiomatic setup allows to preserve the model's simplicity and to focus on the role of abilities' distributional skewness in the outcomes of the game. It also allows to explore more deeply the effect of financial constraints upon the latter outcomes.

Formally, the ability distribution is given by the function

$$p(a) = \begin{cases} \frac{2a}{m}, & 0 \leq a \leq m \\ \frac{2(1-a)}{1-m}, & m < a \leq 1 \end{cases} \quad (2)$$

The moment m , which represents the mode, measures the skewness of the distribution. As long as $m < 1/2$, less able students constitute a larger proportion of the total population. On the contrary, if $m > 1/2$, abler students constitute a larger proportion over the total population. Logically, if $m = 1/2$, the proportion of high ability students equals that of low ability students.

2.3 The University

The university behaving as a monopoly under the proposed setup aims to maximize human capital over the incurred cost to provide a certain level of quality. As in one of the scenarios proposed by [Romero \(2005\)](#), university's surplus positively depends on the human capital production, while negatively on the incurred costs to provide a given level of quality. More specifically, university's preferences are measured by the following

$$U = \int_f^1 \int_{a^*}^1 (q(p(a)) - c(q))dadw \quad (3)$$

where the product-term $qp(a)$ measures human capital production mechanism contingent on university's quality q , where $q \in [0, 1]$, and on the density function of accepted students ([Equation 2](#)); whereas $c(q)$ represents the total cost incurred by the university to provide a certain quality level. Note that incurred costs are assumed to be quadratic in quality, *i.e.*, $c(q) = q^2$. In other words, this implies increasing marginal costs due to resources' scarcity (*e.g.*, professors, classrooms, funds, etc.), and an inelastic supply of such resources.

The bounded structure of quality ensures a bounded structure of university's budget. It means that independently of how university's financial performance will be, the hypothetical government's subsidy, which may aim at keeping the system running, will never exceed the unity measure. However, the role of a possible subsidy is omitted from the analysis. For similar reasons, which include simplicity and more specifically the goal of clearer insights into quality decision, fixed costs are also omitted from the analysis.

As the university strives for the coverage of a market demand composed of individuals, who differ in endowment and ability, the surplus accounting for: (i) the triangular distribution of abilities ([Equation 2](#)); (ii) the available endowment; and (iii) the quadratic costs in quality, can be written as follows

$$\bar{U} = \begin{cases} \int_0^1 \int_{a^*}^m \left(\frac{2a}{m}q - q^2\right) dadw + \int_0^1 \int_m^1 \left(\frac{2a}{m}q - q^2\right) dadw, & 0 \leq a \leq m \\ \int_0^1 \int_{a^*}^m \left(\frac{2a(1-a)}{(1-m)}q - q^2\right) dadw, & m < a \leq 1 \end{cases} \quad (4)$$

Like in [Romero \(2005\)](#), the first integral is allowed to take values along the endowment's full extension $[0, 1]$, informing about the existence of perfect capital markets (PCM), and about the fact that students do not face barriers with respect to education's financing.

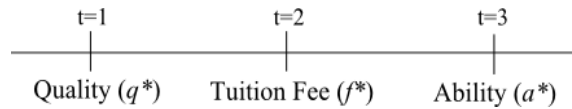
In similar fashion, under borrowing constraints (BC), it will not be possible anymore to sum individuals along the full extension of the endowment. Indeed, considering that individuals will bear the matriculation costs, the endowment integral will take now values

over the restricted segment $[f, 1]$, modelling so education's funding constraints as obstacle. This is formally given by:

$$U = \begin{cases} \int_f^1 \int_{a^*}^m \left(\frac{2a}{m} q - q^2 \right) dadw + \int_f^1 \int_m^1 \left(\frac{2a}{m} q - q^2 \right) dadw, & 0 \leq a \leq m \\ \int_f^1 \int_{a^*}^m \left(\frac{2a(1-a)}{(1-m)} q - q^2 \right) dadw, & m < a \leq 1 \end{cases} \quad (5)$$

However, as f is here allowed to take the zero-value, one can focus in particular on (5) since it fully captures the PCM case (Equation 4).

The university's choice sequence proposed in earlier contributions (Romero and Del Rey, 2004; Romero, 2005) is preserved. The assumption that price decision follows quality decision is in line with specifications of a wide range of IO models with vertical differentiation. As for admission standards, they finalize the sequence given their key role as instruments for fixing pricing deficiencies caused by demand's relevant, but *ex-ante* unobservable characteristics such as students' abilities (Fernández, 1998). The decisions' sequence is illustrated in Figure no. 1. All decision variables take values on continuous and closed intervals $[0, 1]$.



Source: own compilation

Figure no. 1 – Decisions' sequence

3. RESULTS

An equilibrium is a vector, which is composed of three components: quality, tuition fee, and ability threshold, for which: (i) the university maximizes its surplus; (ii) individuals maximize their utility.

Assuming that university will make choices according to decisions' sequence illustrated in Figure no. 1, solutions of the individuals-university game are derived via the backward-induction technique. Solutions will represent subgame perfect *equilibria*.

3.1 Equilibria

The main insight attained from the solution is that the mode (m) of abilities' distribution is an intrinsic part of the subgame perfect *equilibria*. This implies that university's optimal choices are intrinsically dependent on the distribution's skewness.

Results are formalized in the following propositions.

Proposition 1 Under BC, the subgame perfect equilibrium vector is

$$\left\{ q^* = \frac{2}{3m} (2 - \sqrt{4 - 3m}), f^* = 0, a^* = \frac{1}{3} (2 - \sqrt{4 - 3m}) \right\}.$$

Proposition 2 Under PCM, the subgame perfect equilibrium vector is

$$\left\{ q^* = \frac{2}{3m} (2 - \sqrt{4 - 3m}), f^* \leq \frac{2}{9m} (2 - \sqrt{4 - 3m})^2, a^* = \frac{1}{3} (2 - \sqrt{4 - 3m}) \right\}.$$

Proofs are given in Annex.

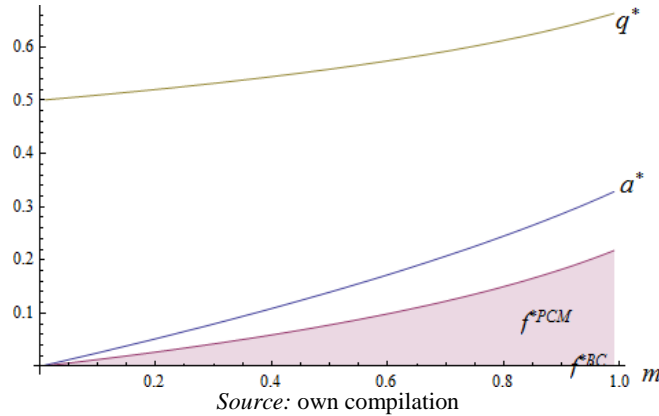


Figure no. 2 – Subgame perfect equilibria and m

Figure no. 2 allows to clearly observe that subgame perfect equilibria bundles are always encountered within a compact and bounded area. The analysis shows that university under PCM may optimally respond by increasing fees as long as distribution's negative skewness increases (*i.e.*, $m \rightarrow 1$). However, such fee levels do not necessarily need to be increasing in m . These may take different values within a closed interval determined as in Proposition 2 and yet preserve their optimality. Additionally, it should be recognized that a human capital maximizing university minimizes by setting to zero tuition fees as long as financial constraints are present. Overall, results indicate that as m varies, the reaction in equilibria qualities is less sensitive than the one in equilibria abilities. Also, it is clear that average ability improvements (exogenously introduced via negative skew) will, in general, contribute to larger equilibrium values for each decision variable.

3.2 Social Welfare

Interestingly, in the proposed setup, the human capital maximizing-type of university symbolically represents an economic agent, whose preferences are well aligned with those of a social planner. However, it must be acknowledged that higher levels of human capital do not necessarily imply improved levels of social welfare. In other words, superior levels of human capital can very well be accumulated at the expense of other welfare dimensions such as equity (*i.e.*, more inclusion through admission of a wider array of abilities) and financial affordability of university education. Hence, this multi-dimensionality is addressed here in order to account for those social welfare components, which are not captured from the one-dimensional feature of the university's objective function (see Equation 3). Consequently, social welfare, here, is specified as a function which positively depend on human capital and inclusion, while negatively on factors such as education's cost and exclusion (*e.g.*, admission rationing). In reality, this specification is supported by several empirical studies (on this see for example,

Morley *et al.*, 2009; Dearden *et al.*, 2011; Caner and Okten, 2013; Radic and Paleka, 2020), which show that raising human capital in the society leads to more economic growth and welfare, whereas raising costs, and imposition of other obstacles to education's access, contribute towards deepening inequalities, and thus undermining social welfare. More formally,

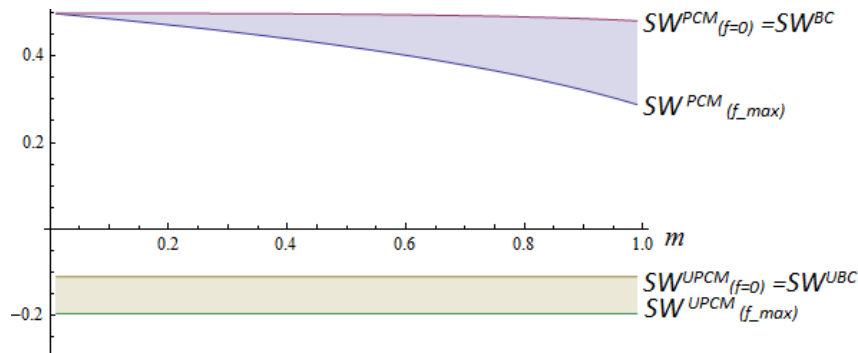
Lemma 1 Social welfare is a function which increases in human capital, and decreases in tuition fee and admission criteria

$$SW = \frac{2q^*}{m} \int_{a^*}^m ada + \frac{2q^*}{1-m} \int_m^1 (1-a)da - \left(1 - \frac{2}{m} \int_0^{a^*} ada\right) f^* - \frac{2}{m} \int_0^{a^*} ada. \quad (6)$$

As results from Propositions 1 and 2 are plugged into Lemma 1, welfare outcomes – under exogenous changes on m and different financial scenarios – are generated. In order to obtain a complete outlook, the corresponding equilibria results reported in Romero (2005) are also plugged into Lemma 1, so the welfare performance with uniformly distributed abilities¹ are discerned too. The latter are denoted by SW^{UPCM} and SW^{UBC} and overall patterns are illustrated in Figure no. 3.

Proposition 3 Regardless of financial constraints, welfare outcomes under triangularly distributed abilities are always superior to those ones generated under uniformly distributed abilities.

Proposition 4 Regardless of whether students' abilities are uniformly distributed or not, welfare outcomes under BC are weakly superior compare to the ones that are generated under the special case of PCM.



Source: own compilation

Figure no. 3 – Social welfare across all scenarios

Proposition 5 The welfare contribution, in the case of triangularly distributed abilities, declines as abilities become more left-skewed, although this contraction is exhibited at different measure. That is, depending on the current state of the capital markets and the exerted pricing policy, when such markets allow for it, SW may fall at higher rates when PCM allow for positive pricing and this is being exerted; whereas it may fall at a very moderate pace when BC are in place and the zero-fee is the only optimal tuition strategy.

3.3 Extension: A Tuition-free System

Within the structure of the model, a tuition-free system would be akin either to a system with BC in place, or to a system with PCM in place when tuition fees are set to zero. In other words, one can imagine a potential cooperative scenario of tuition-free system (*e.g.*, introduced via government regulation) within the proposed non-cooperative structure of the current game as long as fees are set to zero. Interestingly, the model in its proposed fashion allows for such communion between cooperative and non-cooperative choice. In this regard, results suggest that allowing university to select its students' pool does, although not significantly, exacerbate welfare performance when students' skills are triangularly distributed. After all, for a human capital maximizing university a certain level of students' selection may still turn out necessary to be applied in order to preserve an optimal level of human capital production (see Figures 2 and 3). Welfare, however, will not be harmed by selection (see lower part of Figure 3) when potential students' abilities are uniformly distributed.

Overall, it can be affirmed that when non-uniform distributed abilities are present, results here confirm those obtained from Friebel and Maldonado (2015), which point out that allowing universities to select its students within a non-tuition fee setup can be detrimental to welfare. However, the observed symmetry might not be conclusive at all given the fact that their results are drawn out of another setting, with binary types of both, universities and students. Nevertheless, the seemingly symmetric outcomes may, up to a certain extent, complement each other in the sense that initial assumptions about the exhibited scale of monopolistic competition in such markets are of central importance when it comes to derive and determine the most appropriate policies.

4. CONCLUDING REMARKS

The main purpose of this study was to theoretically inquire into the university behavior and welfare implications under the hypothetical conditions of potential students' abilities being non-uniformly distributed. In order to practically achieve that, it was employed a triangular distribution of students' abilities. It became viable, thus, to simulate a wide range of skewness scales, which were introduced via variations on the distribution's mode (m). Additionally, it was assumed that university strives for human capital maximization. Its preferences were then specified under two financial conditions, *i.e.*, perfect capital markets (PCM), and borrowing constraints (BC). As the PCM case was modelled as a specific case of BC, this note made a further step towards the modeling simplicity in this stream of literature.

As the university was assumed to make choices according to a prearranged decisions' sequence, the subgame perfect *equilibria* (SPE) were considered as solution concept of the game. It turned out that SPE depended on the distribution's moment (m) and the current state of financial constraints (PCM or BC). Consequently, it was possible to obtain interesting insights into the optimal university behavior and social welfare. Among other aspects, the following insights should be highlighted.

First, a human capital striving university makes additional efforts to alleviate adverse effects emerging from the presence of non-uniformity in ability distribution. It means that the pool of admitted students will be such as to allow maximization of human capital over the cost incurred to produce it.

Second, as financial constraints impose a real obstacle to individuals' access to education, the university endeavors to internalize negative externalities deriving from them. However, the analysis reveals that such a capability may decline at a higher rate when left-skewed abilities are present and positive pricing policy is carried out (see [Figure 3](#)).

Third, as long as borrowing constraints are faced from potential students, and provided that abilities are triangularly distributed, university's welfare performance will be superior with respect to all alternative scenarios (*i.e.*, PCM and triangularly distributed abilities, BC and uniformly distributed abilities, and, PCM and uniformly distributed abilities). Overall, choosing the pool of students from a population of triangularly distributed abilities is welfare superior compared to a scenario in which students are selected from a population of uniformly distributed abilities.

Fourth, since the proposed model allows for a communion between a hypothetical cooperative scenario of free-university education and a non-cooperative scenario (provided by the proposed setup) in which *equilibria* fees are set to zero, it turns out interesting to verify whether imposing admission criteria would negatively affect welfare. On this point, results here confirm the hypothesis that allowing university to pursue students' selection can be detrimental to welfare as long as triangularly distributed abilities are present and the tuition-free policy prevails.

In conclusion, it is proved that the presence of both hypothetical conditions – non-uniformity in abilities' distribution and borrowing constraints – has a substantial effect on *equilibria* and on welfare performance of a human capital maximizing university. Despite possible pitfalls related to the simplicity of the employed approach, this note may have clarified some theoretical aspects that have not been considered by earlier studies. Concurrently, it may have opened new avenues for future theoretical and empirical research on university behavior. In particular, the usefulness of the assumption of triangularly distributed abilities can be extended to assess other possible higher education market settings (*e.g.*, duopoly, oligopoly). Also, the proposed approach may further motivate the empirical testing of non-uniformity in skills' distribution and the search for real-life scenarios in which potential students' abilities distribution mostly approximates to a triangular pattern. Altogether, these attempts may further shed light on the complex economic relations embodied in the higher education sector of nations where universities have already acquired the capability to self-regulate and have better aligned their missions with benevolent goals such as human capital maximization, equity and satisfaction of a wider spectrum of stakeholders.

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ANNEX

A.1 University's Utility Function

Since PCM is a particular case of BC, it seems more straightforward to directly focus on the latter. Formally,

$$U(a, f, q) = \begin{cases} \int_f^1 \left[\int_{a^*}^m 2q \frac{a}{m} da + \int_m^1 2q \frac{1-x}{1-m} dx - \int_{a^*}^1 q^2 da \right] dw, & 0 \leq a \leq m \\ \int_f^1 \left(2q \frac{1-a}{1-m} - q^2 \right) dadw, & m < a \leq 1 \end{cases}$$

Hence, this generalized form captures a hypothetical case in which the lower limit of the endowment integral is set to zero, *i.e.*, $f = 0$, and this would be akin, in realistic terms, to the scenario of PCM in place. Otherwise, whenever the lower limit of the endowment is allowed to take values higher than zero, *i.e.*, $f \in (0; 1]$ (indicating a reduction of endowment by the corresponding fee burden), the scenario of BC in place emerges. Consequently, the equation above captures all possible scenarios with respect to the state of capital markets, *i.e.*, $f \in [0; 1]$.

Exploiting step by step,

$$U(a, f, q) = \begin{cases} (1-f)q \left[\frac{2}{m} \int_{a^*}^m ada + \frac{2}{1-m} \int_m^1 (1-a)da - q(1-a^*) \right], & 0 \leq a \leq m \\ (1-f)q \left[\frac{2}{1-m} \int_m^1 (1-a)da - q(1-a^*) \right], & m < a \leq 1 \end{cases}$$

Integrating and simplifying the remaining terms, it yields to

$$U(a, f, q) = \begin{cases} (1-f)q \left[\frac{1}{m} a^2 I_{a^*}^m + \frac{2}{1-m} \left(a^* - \frac{a^{*2}}{2} \right) I_m^1 - q(1-a^*) \right], & 0 \leq a \leq m \\ (1-f)q \left[\frac{2}{1-m} \left(a^* - \frac{a^{*2}}{2} \right) I_a^1 - q(1-a^*) \right], & m < a \leq 1 \end{cases}$$

$$= \begin{cases} (1-f)q \left[\frac{1}{m} (m^2 - a^{*2}) + \frac{2}{1-m} \left(\frac{1}{2} - m - m^2 \right) - q(1-a^*) \right], & 0 \leq a \leq m \\ (1-f)q \left[\frac{2}{1-m} \left(\frac{1}{2} - a^* - \frac{a^{*2}}{2} \right) - q(1-a^*) \right], & m < a \leq 1 \end{cases}$$

Since $\left(\frac{1}{2} - z - \frac{z^2}{2}\right) = \frac{1}{2}(1-z)^2$ and $(z^2 - a^{*2}) = (z - a^*)(z + a^*)$, the expression can be simplified into

$$U(a, f, q) = \begin{cases} (1-f)q \left[m - \frac{1}{m} a^{*2} + \frac{2}{1-m} \frac{1}{2} (1-m)^2 - q(1-a^*) \right], & 0 \leq a \leq m \\ (1-f)q \left[\frac{2}{1-m} \frac{1}{2} (1-a^*)^2 - q(1-a^*) \right], & m < a \leq 1 \end{cases}$$

Finally,

$$U(a, f, q) = \begin{cases} (1-f)q \left[1 - \frac{1}{m} a^{*2} - q(1-a^*) \right], & 0 \leq a \leq m \\ (1-f)q(1-a^*) \left[\frac{1-a^*}{1-m} - q \right], & m < a \leq 1 \end{cases}$$

A.2 Equilibria

A.2.1 Optimal a

Solving for optimal ability threshold, utility function is divided into the left-hand side (*i.e.*, $U^l(a, f, q)$) and the right-hand side (*i.e.*, $U^r(a, f, q)$). More formally,

$$U = \begin{cases} U^l(a, f, q) = (1-f)q \left[1 - \frac{1}{m} a^{*2} - q(1-a^*) \right], & 0 \leq a \leq m \\ U^r(a, f, q) = (1-f)q(1-a^*) \left[\frac{1-a^*}{1-m} - q \right], & m < a \leq 1 \end{cases}$$

It is now possible to explore in both parts of utility function. First, as concerns the left-hand side, the following holds:

$\frac{\partial U^l(a,f,q)}{\partial a^*} = (1-f)q \left(q - \frac{2}{m}a^* \right)$ and $\frac{\partial^2 U^l(a,f,q)}{\partial^2 a^*} < 0$ for $0 \leq a \leq m$. Hence, function's concavity and the existence of a maximum at $a^{*l} = mq/2$ is ensured.

Moreover, the value of university's utility at that point is

$$U^l(mq/2, f, q) = (1-f)q \left[1 - \frac{1}{m} \frac{m^2 q^2}{4} - q \left(1 - \frac{mq}{2} \right) \right] = (1-f)q \left(1 - q + \frac{mq^2}{4} \right).$$

On the other hand, the following outcomes are drawn with respect to the right-hand side $\frac{\partial U^r(a,f,q)}{\partial a^*} = (1-f)q \left(q - 2 \frac{1-a^*}{1-m} \right)$ and $\frac{\partial^2 U^r(a,f,q)}{\partial^2 a^*} > 0$. This, of course, enables to affirm that the right-side of utility function is convex and reaches a minimum at the point $a^r = 1 - (1-m)q/2$. Also, $U^r(m, f, q) = (1-f)q(1-m)(1-q)$, so at the point $a^* = m$, the function is continuous, but not differentiable. Also, $U^r(1, f, q) = 0$.

To sum up, it can be affirmed that $a^{*l} = mq/2$ represents the unique maximum of university's utility function. As university's optimal ability choice also depends on the individuals' utility constraint, i.e., $aq \geq f$, the decision on ability threshold, accounting for the latter condition, is formally given by

$$a^* = \begin{cases} \frac{mq}{2}, & f \leq \frac{mq^2}{2} \\ \frac{f}{q}, & f > \frac{mq^2}{2} \end{cases}.$$

A.2.2 Optimal f

In line with the above calculations about optimal ability, university's utility can be formalized as follows

$$U = \begin{cases} U(mq/2, f, q), & 0 \leq f \leq \frac{mq^2}{2} \\ U(f/q, f, q), & \frac{mq^2}{2} < f \leq 1 \end{cases}.$$

By plugging the values of the optimal ability vector in the extended $U^l(a, f, q)$, it yields to

$$U = \begin{cases} U(mq/2, f, q) = (1-f)q \left(1 - q + \frac{mq^2}{4} \right), & 0 \leq f \leq \frac{mq^2}{2} \\ U(f/q, f, q) = (1-f)q \left(1 - q + f - \frac{f^2}{mq^2} \right), & \frac{mq^2}{2} < f \leq 1 \end{cases}.$$

Notice that if the fee is set to zero, then $U(mq/2, 0, q) = q \left(1 - q + \frac{mq^2}{4} \right)$; whereas in the kink point (i.e., $f = \frac{mq^2}{2}$) utility becomes $U(mq/2, mq^2/2, q) = \left(1 - \frac{mq^2}{2} \right) q \left(1 - q + \frac{mq^2}{4} \right)$; and lastly if the fee is set at the maximum (i.e., $f = 1$) $\Rightarrow U(f/q, 1, q) = 0$. Moreover, the left-part of university's utility function is decreasing in f ; whereas the right-part is a cubic function; as a result, it can be inferred the non-monotonicity in f . However, since it is known that fees must comply with the criteria $f > \frac{mq^2}{2}$ the following reasoning can be employed to prove that $U(f/q, f, q) < U(mq/2, f, q), \forall f \in (mq^2/2, 1]$.

If $f > mq^2/2 \Rightarrow 2f > mq^2 \Rightarrow f > mq^2 - f \Rightarrow f^2 > f(mq^2 - f)^2 \Rightarrow \frac{(mq^2)^2}{2} > f(mq^2 - f)$, since $f > mq^2/2$.

Dividing side by side with mq^2 , it yields to $\frac{mq^2}{2} > f \left(1 - \frac{f}{mq^2} \right)$. Adding to each side $1 - q$, it yields to $1 - q + \frac{mq^2}{2} > 1 - q + f \left(1 - \frac{f}{mq^2} \right)$. Since $f > mq^2/2$ it is also expected that the following will

hold $\frac{\left(1 - \frac{mq^2}{2} \right)}{(1-f)} \left[1 - q + \frac{mq^2}{2} \right] > 1 - q + f \left(1 - \frac{f}{mq^2} \right)$, which can be finally simplified into $\left(1 - \frac{mq^2}{2} \right) \left[1 - q + \frac{mq^2}{2} \right] > (1-f) \left[1 - q + f \left(1 - \frac{f}{mq^2} \right) \right]$.

It is thus proved that $U(f/q, f, q) < U(mq/2, f, q)$. Such a result implies that university's utility is maximized when tuition fee is set to zero. Indeed, the maximum utility level is achieved under BC,

and its value is $U(mq/2, f, q) = q \left(1 - q + \frac{mq^2}{4}\right)$; whereas under PCM, which according to the employed setup is considered as a specific BC scenario, the fee can be set within the interval $\left[0, \frac{mq^2}{2}\right]$ and yet preserve its optimality.

A.2.3 Optimal q

Considering the former results about optimal ability and fee, the utility function is simplified into $U(mq/2, 0, q) = q \left(1 - q + \frac{mq^2}{4}\right)$.

As PCM is a very specific case of BC, optimal quality should reflect the generalized zero-fee policy coherent with a continuous spectrum of possible financial scenarios.

The first and second derivative with respect to q are

$\frac{\partial U}{\partial q} = \frac{3}{4}mq^2 - 2q + 1$; and $\frac{\partial^2 U}{\partial^2 q} = \frac{3}{2}mq - 2 < 0$. Consequently, the viable solution is

$q^* = \frac{2}{3m}(2 - \sqrt{4 - 3m})$. Accordingly, the perfect subgame equilibrium for the generalized BC case is $\left\{q^* = \frac{2}{3m}(2 - \sqrt{4 - 3m}), f^* = 0, a^* = \frac{1}{3}(2 - \sqrt{4 - 3m})\right\}$. As for the PCM case, which is previously defined as a specific BC scenario, the perfect subgame equilibrium is $\left\{q^* = \frac{2}{3m}(2 - \sqrt{4 - 3m}), f^* \leq \frac{2}{9m}(2 - \sqrt{4 - 3m})^2, a^* = \frac{1}{3}(2 - \sqrt{4 - 3m})\right\}$.

A.3 Social Welfare

The SW function laid out in Lemma 1 is integrated and simplified as follows

$$\begin{aligned} SW &= \frac{2q^*}{m} \int_{a^*}^m ada + \frac{2q^*}{1-m} \int_m^1 (1-a)da - \left(1 - \frac{2}{m} \int_0^{a^*} ada\right) f^* - \frac{2}{m} \int_0^{a^*} ada \\ &= \left(m - \frac{a^{*2}}{m} + 1 - m\right) q^* - \left(1 - \frac{a^{*2}}{m}\right) f^* - \frac{a^{*2}}{m} \\ &= \left(1 - \frac{a^{*2}}{m}\right) q^* - \left(1 - \frac{a^{*2}}{m}\right) f^* - \frac{a^{*2}}{m} + 1 - 1 = \left(1 - \frac{a^{*2}}{m}\right) (q^* - f^* + 1) - 1 \end{aligned}$$

Hence, SW in a simplified form can be written as

$$SW = \left(1 - \frac{a^{*2}}{m}\right) (q^* - f^* + 1) - 1.$$

Notes

¹ By referring to that paper, SPE are the following: $SPE^{PCM} = \left\{c'(q^*) = \frac{1+a^*}{2}, f^* \leq c(q), a^* = \frac{c(q)}{q}\right\}$, $SPE^{BC} = \left\{c'(q^*) = \frac{1+a^*}{2}, f^* = 0, a^* = \frac{c(q)}{q}\right\}$. Moreover, if quadratic costs are assumed, *i.e.*, $c(q) = q^2$, it yields to $SPE^{PCM} = \left\{q^* = \frac{1}{3}, f^* \leq \frac{1}{9}, a^* = \frac{1}{3}\right\}$, $SPE^{BC} = \left\{q^* = \frac{1}{3}, f^* = 0, a^* = \frac{1}{3}\right\}$. By plugging the previous *equilibria* values into Lemma 1, it implies that: $SW_{(max_f)}^{PCM} = -\frac{5}{27}$, $SW_{(f=0)}^{PCM} = -\frac{1}{9}$, $SW_{(f=0)}^{BC} = -\frac{1}{9}$.

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