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Exploring Hedging Strategies Identified by Fractal Dimensions

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Abstract

A hedging strategy is designed to increase the likelihood of desired financial outcomes. Market speculators hedge investment positions if they are worth protecting against potential negative outcomes on the underlying investment. Such negative outcomes cannot be avoided altogether, but effective hedging can reduce impact severity. The investment strategy includes an index held by investors (long position) and uses a fractal dimension indicator to warn when liquidity or sentiment changes are imminent. When the named indicator breaches a certain threshold, a hedging position is taken. This sequence of events triggers the implementation of a hedging strategy by entering a buy put-option position. The daily cumulative returns on using the fractal dimension indicators were 83% more profitable on average when applied to each chosen index respectively.

Keywords: fractal market hypothesis; hedging strategy; trading strategy.

JEL classification: C52; G11.

1. INTRODUCTION

Investment strategies involve assembling portfolios based on the behaviour, knowledge, insight and skill of the investor. The construction of these investments is driven by a checklist of investment goals: outperform the benchmark, minimise the portfolio variance, maximise the absolute return, etc., to satisfy the investment goals (Crapo, 1999).

Hedging strategies are used in conjunction with investment strategies to ameliorate the impact of market risk present in the investment. Hedging strategies thus protect investor's portfolios values in volatile or uncertain market conditions. Financial derivatives are usually employed for hedging strategies to limit losses brought about by volatile market conditions. Through derivatives, investors invoke a type of insurance on investment portfolios, but such transactions are not free (Lewellen *et al.*, 1977).

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Misalignment of multiple investment objectives (which include risk and return goals, disparate investment horizons, etc.) lubricates financial markets, giving rise to liquidity in efficient markets. This is good news for market participants; the ability to trade large asset volumes quickly by pooling large numbers of buyers and sellers who think and trade differently, without impairing the market price, is highly desirable (Vigna and Haberman, 2001; Borio and Lowe, 2002).

The weak form of the Efficient Market Hypothesis (EMH) asserts that, when all facts are publicly available in the market high price volatility is impossible. This is not true in practice: large volatility spikes are a characteristic of most financial markets. This demonstrates that the EMH is fundamentally flawed: the EMH assumes that investors are a homogenous group of thinkers with the same investment horizon (Malkiel, 2003).

An alternative to the EMH is the Fractal Market Hypothesis (FMH), which incorporates the differences between investment horizons of market participants (Fama, 1998). The FMH is fundamentally different to the EMH in that it recognises that investors have different time horizons also interprets and evaluates the same information and facts differently.

When all investors are actively involved in the market with different time horizons, a rich fractal structure within the market arises. This environment optimises liquidity and price stability. When group-thinking and their time horizons converge amongst investors, however, liquidity evaporates and severe destabilisation results (Kristoufek, 2012).

The fractal dimension of the time series of an asset is a geometric dimension, which has been found to flag the onset of chaotic behaviour when a given threshold is breached. When fractal dimensions decrease and approach 1, increasing group-think is indicated with a corresponding decrease in liquidity (Karp and van Vuuren, 2019).

This study explores the performance of an investment strategy comprising an index and subsequently a hedging strategy whose implementation is triggered by the value of the index's changing fractal dimension. This strategy is tested under different market conditions and in different geographies. The principal variables are the fractal dimension of the investment (the index) which breaches in a user-defined (but market-calibrated) threshold when group-think arises, thereby triggering the implementation of a hedging strategy to prevent losses. Essentially, the fractal dimension acts as an early warning indicator.

This remainder of this article proceeds as follows: A literature review is presented in Section 2 which provides insight into the fractal dimension indicator and previous work on the topic. The fractal dimension is a novel concept and has not been the focus of fevered academic investigation to date, so literature on the subject is somewhat limited to generic findings and performance measurement. The data used in this study and the methodology used (including the mechanics of the fractal dimension indicator) are set out in Section 3. The results obtained are presented and discussed in Section 4. Section 5 concludes.

2. LITERATURE REVIEW

This study is motivated by the fact that the FMH considers different time horizons which exist for large number of different investors with different investor preferences and requirements. In Figure no. 1, an index price is simulated, and the corresponding fractal dimension represented below on the same timescale. When the fractal dimension breaches a selected threshold (shown by ellipsis) hedge strategies are implemented (H_1 through H_6). Arrows indicate the initiation dates of option implementation.

When investment horizons converge (i.e. group-think arises and large swathes of investors behave similarly) available liquidity is strongly affected. The fractal dimension decreases in these periods towards 1 and, on breaching a threshold of 1.25, triggers implementation of a strategy to hedge against negative outcomes from an impending liquidity crisis and associated price falls. The reduction of liquidity (and price decrease associated therewith) within the financial market is the principal driver of the fractal dimension indicator. It is the price decrease that long investors wish to limit (Anderson and Noss, 2013).

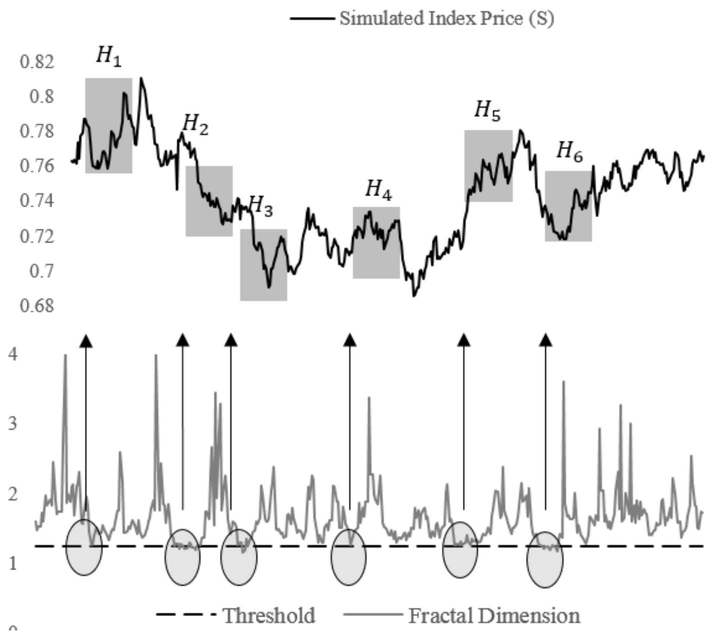

Figure no. 1 – Simulation showing hedge strategy timing.

Breaches are not indicators of price movement severity, but rather for changes in underlying asset price liquidity (Kristoufek, 2013). Figure no. 2 shows the simulated index price with the fractal dimension implemented on this price. The first breach of the fractal threshold is shown by B_1 and the according duration of the hedging strategy by the grey area above shown in H_1. This simulation, in Figure no. 2, shows the application method in detail on each breach of the fractal dimension.

Investors alerted to this imminent illiquidity will wish to limit the associated downward surge in price by hedging these positions using derivatives (Triantis, 2005). For this study the simulation involved investors installing an option, a put option (Agarwal and Naik, 2000). Purchasing a put option allows investors to hedge the long position of the index in the investment portfolio. This hedging strategy provides investors with the right (but not the obligation) to sell the current position in the index (long position) at a specific price within a given period (Merton *et al.,* 1982). Also, for this study, the option is held from where the fractal dimension threshold was first breached, indicating a change in liquidity within the asset's attractiveness until the fractal dimension is breached again indicating an opposite change in liquidity.

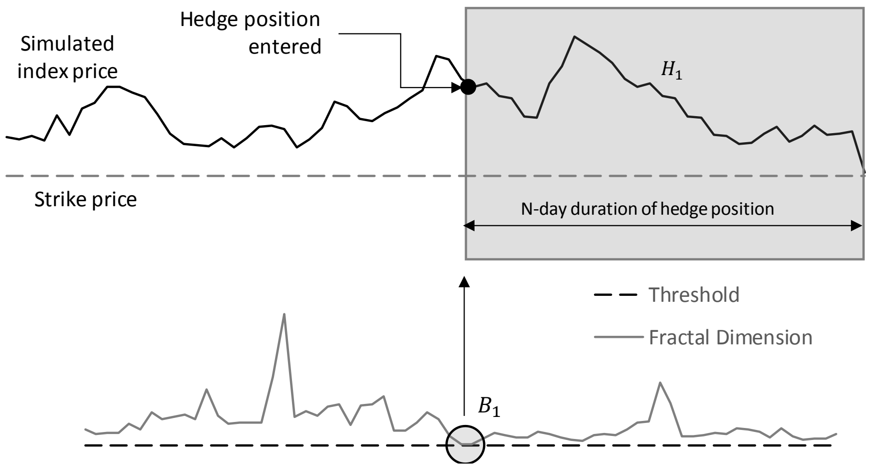

Figure no. 2 – First fractal breach with duration of hedge strategy

By limiting the downside risk of the investment strategy, investors will aim to offset losses on long positions (of the hedge position) by buying and holding put options between breaches to generate returns on the investment. The success of the investment strategy is governed by the relationship between the long position and the option price in the given time span. The put option's price will generally decrease as the underlying asset's price increases and *vice versa* (Bunch and Johnson, 2000). Another factor that will impact the success of the hedge strategy is the time the put option is held by investors or the time till expiration which is driven by the volatility of the underlying asset. If a high volatile environment is entered investors could expect large movement in price before the expiration of the put option, and when the option is held for a relative long period (determined by the breach of the fractal dimension) the higher also will be the option price. This second factor can also impact the success of the investment strategy (Ni *et al.*, 2005).

The fractal indicator will be applied and observed as follows: by using the relative daily share price data over the full chosen time period the fractal dimension is determined for each day. This fractal dimension is evaluated against a chosen threshold of 1.25 since there is not a given pre-determined threshold given to implement a fractal dimension. The threshold is chosen by investors and may be tested at different thresholds. By comparing the fractal dimension and the chosen threshold of each chosen index separately the fractal dimension is monitored for a breach of the chosen threshold i.e. when the fractal dimension is lower than the threshold, this is considered a breach.

Once the threshold is breached, imminent large movements in the price of the index are possible, but investors does not know in which direction the movement will be. The breach indicates the implementation of the hedge strategy by buying a put option, this limits the downside risk for investors by providing the option (but not the obligation) to sell the index at the strike price. The pricing of the put option is derived using the Black and Scholes formula.

When the breach is indicated, and a hedge strategy is in place the investment faces one of two outcomes. When the duration of the put option has reached maturity the index price will either be below or above the strike price. If the index price is below the strike price investors will exercise the put option and sell the index at the strike price, shown in Figure no. 3.

Investors then have limited downside risk to price changes. The other outcome is where the index price has increased above the strike price when the put option has reached its maturity, indicated by Figure no. 4. In this scenario investors also have the right to exercise the put option but will choose not to do so. In this case investors will retain the long position on the index and only incur the loss of paying the premium to purchase the put option.

Investors have the optimal scenario when they have purchased the put option and at maturity the index price has increased by a significant percentage above the strike price; enough to offset the premium cost and, in some cases, enough to make considerable gains. Investors then repeat this strategy over the period to investigate what the daily cumulative return would be by comparing the strategy performance with index performance.

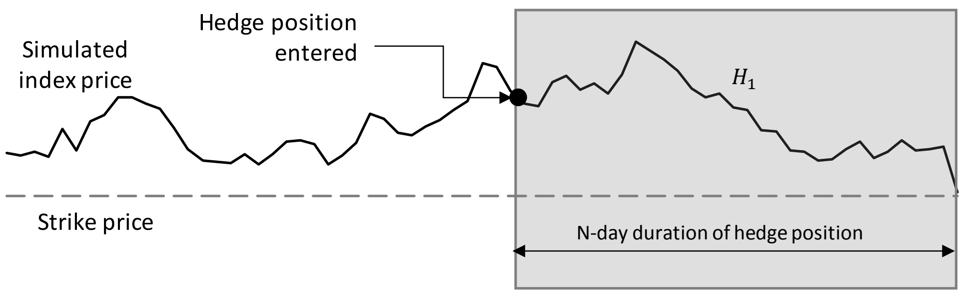

Figure no. 3 – Index price is above strike price at maturity of hedge strategy

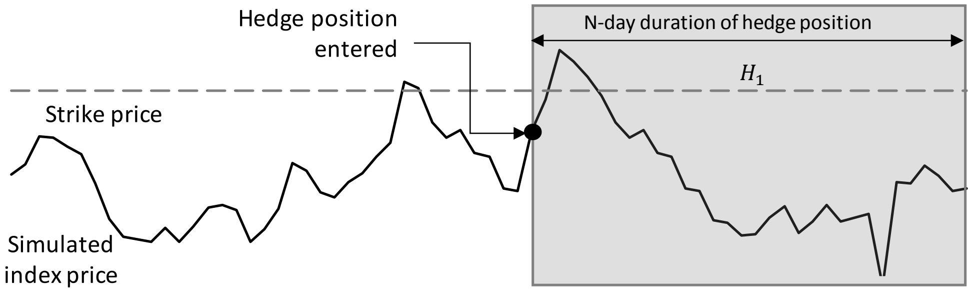

Figure no. 4 – Index price is below strike price at maturity of hedge strategy

An important component to ensure accurate implementation of the strategy is the timing between holding the investment portfolio (long position) and implementing the fractal dimension indicator on the long position and subsequently entering the hedging strategy (Miller and Folta, 2002). The hedging strategy is then maintained for the period until the fractal dimension again breaches the threshold for a second time indicating a change in the attractiveness of the underlying asset. This indicates when the put option must be exercised. After the put option is exercised investors again enter long positions in the underlying asset (indicated by the second breach of the fractal dimension threshold). This process is repeated for the duration of the selected time period to establish of the FMH is a viable indictor for market liquidity and whether the fractal dimension is a valid indicator for a change in liquidity in the right direction in favour of investors.

3. DATA AND METHODOLOGY

3.1 Data

The data used in this report are market index values which span a 15-year period from January 2004 to December 2018. This period was chosen to include relatively low market volatility (Jan 2004 to Jun 2011), the highly volatile financial crisis of 2008/9 and another period of high market volatility (Jul 2011 to Dec 2018) and corresponding growth. Stock indices from three different emerging market stock exchanges were chosen for comparison:

- the Johannesburg Stock Exchange All Share Index (JSE ALSI): A South African equity index containing top listed companies weighted by factors such as size and liquidity, the index includes some 150 different JSE-Listed companies and is calculated in ZAR. The JSE is the 19th largest stock exchange and largest by market capitalisation in Africa (JSE, 2019b). Adding an African all share index which reflects the equity market movements of South Africa, provides diversification to the data pool and,
- the Bovespa Index (better known as the Ibovespa) traded on B3 (located at São Paulo, Brazil) and is also denoted in Brazilian Real. This index is compiled from 60 stocks listed in the B3 (Ibovespa, 2019) and serves as a benchmark for the Brazilian market, classified as an emerging market and,
- the Russian Trading System Index (RTSI) is an index compiled by 50 Moscow Exchange listed stocks calculated on a free-float capitalisation-weighted index basis. This index is calculated in USD and revaluated every three months. This index is also a benchmark index for the Russian financial market is classified as an emerging market (RTSI, 2019).

These three indices were selected as benchmarks for emerging markets (Bannier *et al.*, 2019).

The volatility for each index was calculated by using the exponential weighted moving average (EWMA) volatility of the respective volatility index. The JSE's South African Volatility Index (SAVI) is used for the JSE All Share Index (JSE, 2019a), modelled on the Chicago Board Options Exchange's (CBOE) Volatility Index (VIX). The VIX was also used for the Brazilian Index (CBOE, 2019). The Russian Volatility Index (RTSVX) was used as the volatility measure for the Russian Trading System (RTSVX, 2019).

3.2 Methodology

JP Morgan (1996) describe the EWMA model which uses a combination of historical and implied volatility forecasts. This uses an exponential moving average to give extra weight to more recent historical values when calculating the volatility:

$$\sigma_t = \sqrt{\lambda\sigma_{t-1}^2 + (1-\lambda)r_t^2} \quad (1)$$

where σ_t is the volatility at time t, λ is the decay factor, $t-1$ is the volatility for the previous period and r_t is the percentage return over the current period, for which the volatility is being calculated. Average annual dividend yields were collected from respective index data providers. Risk-free rates are those commonly used in the countries where the index operates. The three risk-free rates used for each index respectively are: the Johannesburg Interbank Agreed Rate (JIBAR) in South Africa and in Brazil the 3-month government bond yield rate was used (Brazil 3-month Bond Yield), lastly the risk-free rate used for the Russian Index was the Russia 3-month bond rate (Russia 3-month bond yield).

All option maturities, T, were three months. Although other maturities are possible in principle, these are uncommon in practice (Bartoňová, 2012). Dividends affect option prices and to account for this, they are removed from the index level, S_0 using:

$$D = S_0 \alpha e^{-rT} \tag{2}$$

where α is the annual dividend expressed as a percentage and D is the currency amount of the relevant dividend.

Nominal index levels were calculated to give comparable, standardised representations, using S_t/S_0 where S_t is the current stock price and S_0 is the initial stock price at which the put option was instituted. Put option prices for the hedging strategy are calculated using:

$$p = K_p e^{-rT} N(-d_2) - S_0 N(-d_1) \tag{3}$$

where K_p is the put strike price, r is the risk-free rate of return, S_0 is the stock price, T is the time to expiration, $N(x)$ is the normal distribution density of x, and d_1 and d_2 are given by:

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$

and

$$d_2 = \frac{\ln\left(\frac{S_0}{K}\right) + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

respectively, where K is the strike price for the put or call, σ is the stock volatility and T is the time remaining to maturity (Black and Scholes, 1973). S_0, σ, r and α are obtained from historical, empirical data. The i^{th} 1-day log return, is:

$$r_i = \ln\left(\frac{P_i}{P_{i-1}}\right) \tag{4}$$

where P_i is the i^{th} day's index price. The scaling factor, n (the number of days a fractal dimension should return (Joshi, 2014a)) is used to determine the i^{th} day's fractal dimension, $D_{i,n}$, with scaling factor n:

$$R_{i,n} = n - day \text{ log return on day } i = \ln\left(\frac{P_i}{P_{i-n}}\right) \tag{5}$$

$$N_{i,n} = \text{scaled return on day } i \text{ with scaling factor } n = \frac{\sum_{i-n}^{i}(abs\ r_i)}{\left[\frac{abs(R_{i,n})}{n}\right]} \tag{6}$$

$$D_{i,n} = \text{fractal dimension on day } i \text{ with scaling factor } n = \frac{\ln(N_{i,n})}{\ln(n)} \tag{7}$$

4. RESULTS

The 15-year time period used to perform the analysis includes turbulent conditions as well as calm market periods in each market at each location. Figure no. 5 shows the three sections examined. Period 1 (Jan 2004–09) is characterised by strong, persistent increases in each index nominal value. The impact of the financial crisis of 2008/2009 is evident.

In Period 2, all indices show signs of recovery. This period reflects a trading phase rather than a trending phase as no new peaks are reached. In Period 3, clear distinctions arise in the movements for each index. The Bovespa Index and RTSI are highly correlated from Jan 2013 to Mar 2014, then the Bovespa outperforms the RTSI, reaching its peak around 380% in Dec 2018. The JSE ALSI increases considerably until reaching its peak in Jan 2018 of approximately 600%.

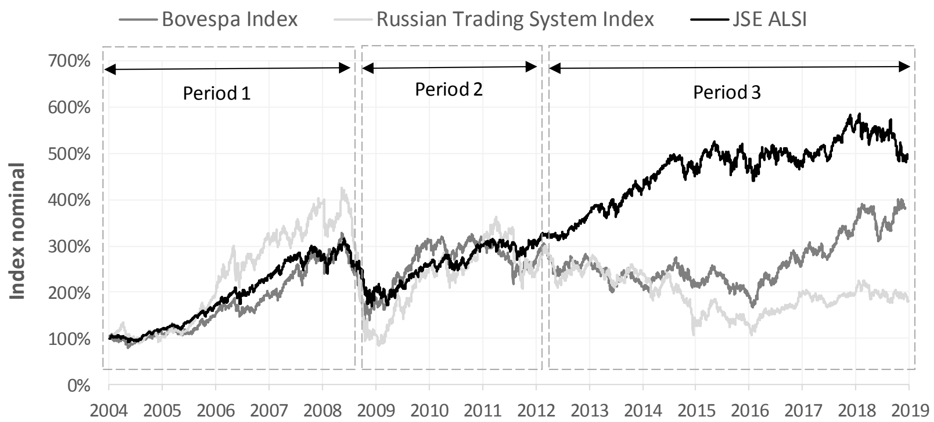

Figure no. 5 – Rebased value of each index over chosen time period. Jan 2004 = 100%

Figure no. 6 shows the fractal indicator applied to the RTSI. There are clear breaches of the 1.25 threshold: each breach signals the implementation of the hedging strategy.

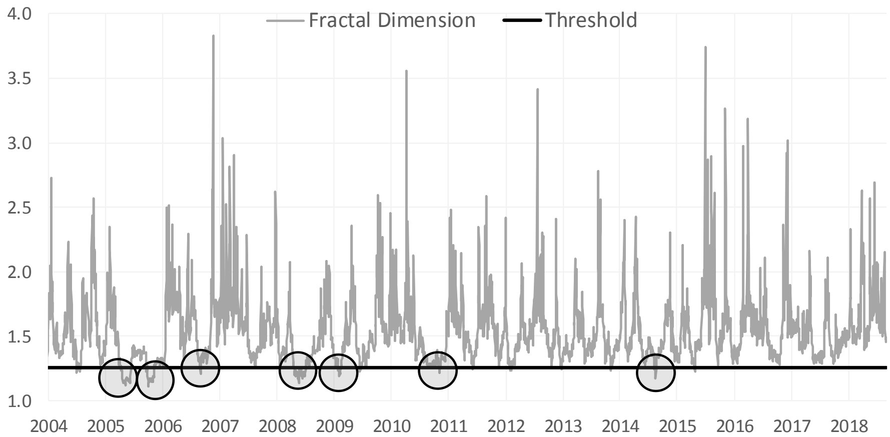

Figure no. 6 – Fractal dimension of the RTSI showing breaches

Each index on each own exhibited different behaviour regarding the number of breaches of the chosen threshold of the fractal dimension at 1.25. The Ibovespa Index had few breaches (27) from 2004-2016 while the JSE ALSI (77) and RTSI (179) reached their peaks during the turbulent market conditions between 2004 and 2009. The number of breaches per index during this turbulent time period shows that if more breaches indicated by the fractal dimension is needed to limit the downside risk of holding the investment in a long position. More hedging positions were implemented for the Ibovespa Index between 2016 and 2018 whereas the JSE ALSI and RTSI had more hedging positions between 2004 and 2009. Thus, there were more volatile market price movements for the latter indices and their downside risks could be hedged using a put option.

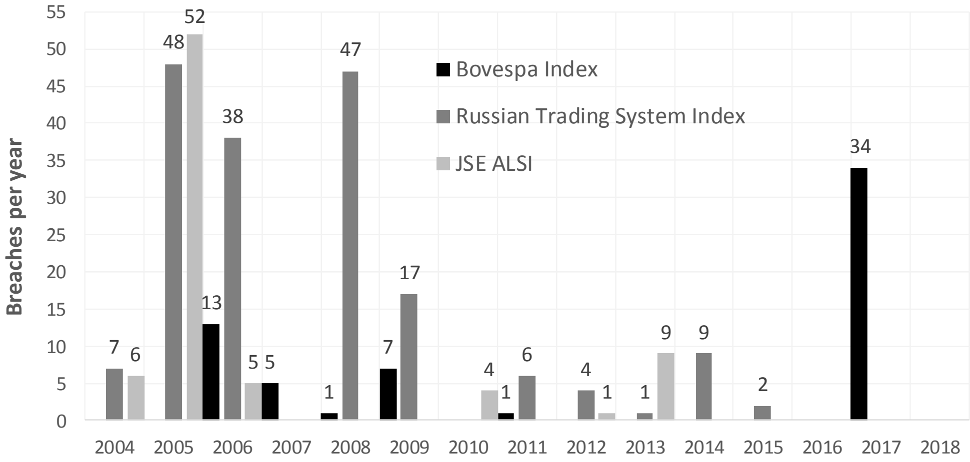

Figure no. 7 – Number of annual breaches for each index

There are clear differences in each index's performance. The hedge strategy outperformed for the Ibovespa Index and RTSI while for the JSE ALSI, it underperformed. Since the JSE ALSI witnessed a surge in breaches in 2004–05, it is assumed that this factor also impacted the strategy performance by not indicating more breaches after 2005 the JSE ALSI did not have the opportunity to limit the downside risks. The JSE ALSI had more breaches followed by increases in index price where the strategy was implemented. If the index price is higher than the put strike price, investors will hold the long index position and only lose the premium paid for purchasing the put. The fractal dimension increased slightly for the JSE ALSI in 2010–13 whereas the Ibovespa Index and RTSI experienced several declines and subsequent breaches in 2011-2015 and 2017 respectively. This comparison, shown in Table no. 1, indicates that the JSE ALSI had fewer large price movements when compared to the price increases of the Ibovespa Index and RTSI, the JSE ALSI had 16% on average per year to ultimately increase by the largest percentage over this period. The Ibovespa index and RTSI increased on average by 9.1% and 8.0% respectively. The cumulative return gained by the Ibovespa Index and RTSI during the time period of 2010 to 2016 was enough indicated by the fractal dimension was enough to outperform their respective index whereas the JSE ALSI did not accumulate enough returns in order to set off the return from the JSE ALSI itself.

We also compare the economic mean of the selected indices in Figure no. 8 and note that the same conclusion may be drawn when compared with the results from Figure no. 7. The economic mean is higher in 2005, 2008 and 2017. These relative higher means may be influenced from the number of breaches from the RTSI and JSE ALSI in 2005, the RTSI in 2008 and the Ibovespa Index in 2017.

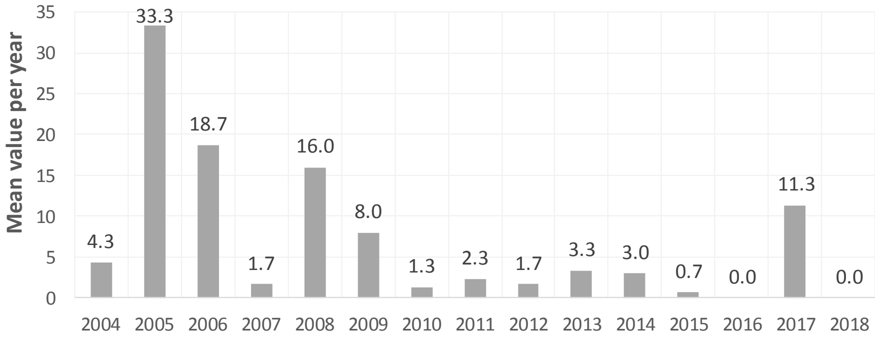

Figure no. 8 – Economic mean of selected indices

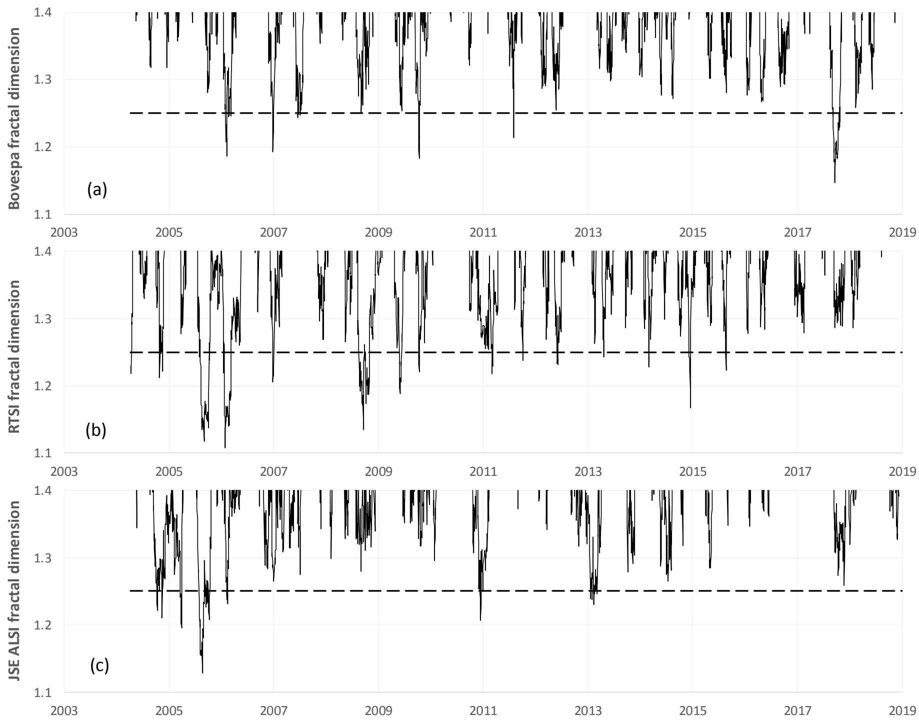

Figure no. 9 – Fractal dimensions and breaches for each index

Figure no. 8 indicates that the number of breaches of the fractal indicator per index during this turbulent time (the financial crisis). This in turn shows that more breaches identified by the fractal dimension are needed to limit the downside risk of holding the investment in a long position. Relative to the other years more breaches were hedged in the years 2005, 2006 and 2008 which may be attributed to the volatile index movement brought about by the financial crisis. More volatile market conditions constituted to more hedged position opened during these time periods.

Table no. 1 – Cumulative daily returns

Ibovespa Index		Russian Trading System Index		JSE ALSI	
Index	Hedge strategy	Index	Hedge strategy	Index	Hedge strategy
296%	337%	95%	365%	380%	320%

A comparison of the risk taken by each index and implementing the hedge strategy with different durations for which the put option is held shows that risk increases as the duration of the put option increases for each index (Figure no. 10). The volatility of the Ibovespa Index increases as the duration for the put option increases but reaches an optimal point around 135-days whereas the RTSI and JSE ALSI also increases over the duration of the put option and ultimately is still increasing as the duration increases over the chosen durations. Relative to the other indices, the JSE ALSI exhibited lower levels of risk undertaken over the chosen period and in combination with the observation made where the JSE ALSI's daily cumulative return did not outperform the applied hedge strategy due to fact that it was also observed that the JSE ALSI had the least number of breaches after 2009 and also considering the nominal growth was relatively lower that the Ibovespa Index and the RTSI.

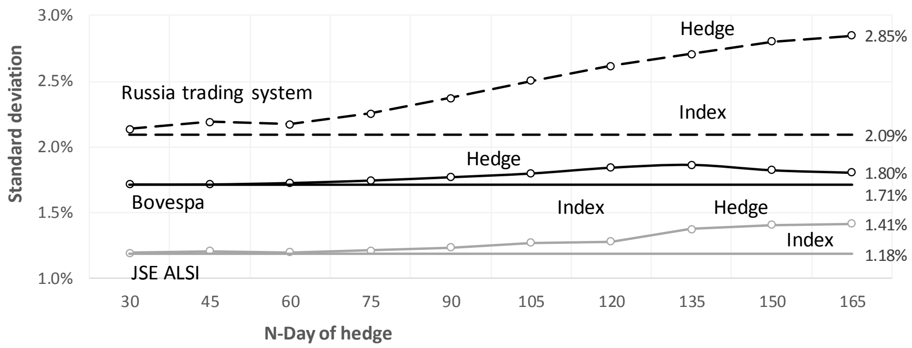

Figure no. 10 – Indices' standard deviation over different hedge horizons

When there is substantial disagreement among financial market participants the fractal dimension is less accurate in forecasting large price movements. This scenario is in the forefront for market liquidity and reflects a quantitative demonstration of the level of market liquidity at a given time. Holding a long position in the index, estimating the daily fractal dimension and implementing the hedging strategy only when breaches occur, could limit downside risk for investors.

For the case where the strategy applied to the Ibovespa Index and RTSI, both outperformed their respective index performance showed by daily cumulative returns. However, in the case of the strategy applied to the JSE ALSI the index outperformed the applied hedge strategy. Deeper exploration shows that the JSE ALSI had relatively small nominal growth when compared to the Ibovespa Index and RTSI. Also contributing to this anomaly is the amount of breaches the JSE ALSI experienced over the time period. All three indices had breaches during the financial crisis of 2008/2009, but where the Ibovespa Index and RTSI still showed several breaches thereafter, the JSE ALSI's number of breaches decreased and had the second number of breaches in total. The combination of these factors in conjunction are the contributing factors to implement the strategy in a successful manner that could be profitable to investors.

5. CONCLUSIONS AND RECOMMENDATIONS

This article explored the implementation of a fractal indicator with a predetermined threshold combined with an option-based hedging strategy for three different geographical locations Option purchases were triggered to limit downside risk when the fractal dimension threshold was breached. The fractal indicator was found to be effective when applied to two of the three tested indices. When the hedging strategy was applied to the Ibovespa Index and RTSI the daily cumulative returns were 337% and 365% respectively, while if the hedging strategy was not implemented the daily cumulative returns would have decreased to 296% and 95% respectively. For further observation the same strategy could be applied to developed countries also and could then be compared to developing countries to establish if the strategy had a more promising application to a developing – or developed countries. This performance measurement could indicate that if the EMH does not hold within the financial market, the FMH may be used as an alternative view of market sentiment. Motivating this view is the impact of implementing a strategy to take advantage of the proposed strategy.

Furthermore, the threshold selected, 1.25, has been empirically determined. For fractal dimensions less than this value, herd behaviour ensues, and liquidity evaporates. This threshold could potentially be optimised for each asset or asset class. In addition, for this work, indices (comprising large numbers of stocks) were selected for applying the hedge strategy. Future work could investigate whether the strategy should be applied to a single selected stock or to different asset classes (such as gold – see Joshi, 2014b) and compare the profitability of each chosen scenario separately.

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