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Ranking Alternatives by Pairwise Comparisons Matrix and Priority Vector

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Abstract

The decision making problem considered here is to rank *n* alternatives from the best to the worst, using information given by the decision maker(s) in the form of an $n \times n$ pairwise comparisons (PC) matrix. We investigate pairwise comparisons matrices with elements from a real interval which is a traditional multiplicative approach used in Analytic hierarchy process (AHP). Here, we deal with two essential elements of AHP: measuring consistency of PC matrix and the method of eliciting the priority vector by which the final ranking of alternatives is derived. Classical approaches introduced by T. Saaty in AHP are compared with later approaches based on the AHP criticism published in the literature. Advantages and disadvantages of both approaches are highlighted and discussed.

Keywords: pairwise comparisons matrix; priority vector; ranking alternatives; analytic hierarchy process; AHP.

JEL classification: C44

1. INTRODUCTION

Recently, each entrepreneur has its' own personal computer(s), tablets, mobile phones, or other modern information technology means. Moreover, there is an increasing popularity of methods for decision support solvable by the help of computers. Multiple Criteria Decision Methods (MCDM) proved to be useful methods e.g. in the following areas:

- buying equipment (cars, machines, furniture),
- investment opportunities,
- services evaluations, etc.

Pairwise comparisons (PC) method is based on the psychological observation that the human brain cannot compare more than 5 to 9 independent values in one moment (so called „cognitive overload"). For a human being it is much easier to compare only two elements: to do pairwise comparisons for all pairs, i.e. pair-wise comparison matrix (relation) *A* which is usually reciprocal.

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Recently, pairwise comparisons often identified with Saaty's AHP. On one hand, AHP is praised in many practical applications, however, on the other hand, it is still considered by many authors as a flawed method that could produce controversial rankings. In this paper we analyze two elements of PC method (a part of AHP) in order to remove theoretical problems of the original method and thus support better decisions. Particularly, we deal with two key elements of AHP: measuring consistency of PC matrix, and eliciting the priority vector by which the final ranking of alternatives is derived. Classical approaches introduced by T. Saaty in AHP, see e.g. Saaty (1991), are compared with later approaches based on the AHP criticism published in the literature, see Bana e Costa and Vansnick (2008), Whitaker (2007). Here, advantages and disadvantages of both approaches are highlighted and discussed.

As a solution of the DM problem, i.e. the best alternative(s), ordinal ranking of the alternatives is required, however, very often, an ordinal ranking among alternatives is not a sufficient result and a cardinal ranking called here "rating" is required.

Some well-known limits to our brain capacity to handle several alternatives at a time is known (it is called the cognitive overload). It makes it impossible to obtain the rating by a priority weighting vector directly, for instance by asking the DM to provide the utility values for the alternatives. A more appropriate and sometimes easier approach is to ask the DM for his opinion over the pairs of alternatives and then, based on the acquired information over the pairs, to derive the rating of the alternatives. A popular mathematical tool for eliciting the expert's preferences by pairwise comparisons between the individual alternatives is the pairwise comparison matrix, the procedure is usually called the pairwise comparison method.

Probably, the first human who wrote about PC method was a medieval monk and scholar Ramon Llull. Later on, the method was mentioned by Marquis de Condorcet in 1785, and was explicitly mentioned and analyzed by Gustav Fechner in 1860, made popular by Luis Thurstone, in 1927, and was transformed into a kind of formal methodology by Thomas Saaty in 1977 (called AHP - *Analytic Hierarchy Process*). Pairwise comparisons have been widely used also in well-known decision making approaches, such as PROMETHEE method, TOPSIS and many others, see e.g. Greco *et al.* (2016). A large number of MCDM methods deriving a ranking/rating of the alternatives have been proposed in the literature, see e.g. Greco *et al.* (2016). In this paper we shall investigate two well-known approaches, particularly, the eigenvector method (EVM) known from AHP (Saaty, 1991), and the geometric mean method (GMM), being in fact the Logarithmic Least Squares Method (LLSM) (Barzilai, 1997).

In one of the most popular MCDM method - the above mentioned Analytic Hierarchy Process (AHP), (Saaty, 1977), the decision problem is structured hierarchically at different levels, each level consisting of a finite number of elements. The DM searches for the priorities representing the relative importance of the decision elements at each particular level. By suitable aggregation he/she finally calculates the priorities of the alternatives at the bottom level of the hierarchy. These priorities are interpreted with respect to the goal at the top of the hierarchy, and then elements at upper levels such as criteria, sub-criteria, etc. The elicitation process at given level is performed by pairwise comparisons of all elements at given level of the hierarchy with respect to the elements of the upper level. If he/she prefers so, the DM may directly use a numerical value from the scale to express the ratio of elements' relative importance. Inserting appropriate values into the given positions a PC matrix is composed. Now, the role of prioritization method is to extract the relative priorities - weights of all compared alternatives.

The values representing the preferences of the decision elements - the alternatives can be also considered as the results of aggregation of pairwise comparisons of a group of decision makers and/or experts. Then the DM problem becomes the group DM problem (GDM). The tournament ranking problem is another well-known application of pairwise comparisons.

The organization of this paper is set out as follows. In Section 2, we introduce main properties of PC matrix. In Section 3, we deal with two essential elements of AHP: measuring consistency of PC matrix and eliciting a priority vector by which the final ranking of alternatives is settled. Classical approaches introduced by T. Saaty in AHP will be compared with later approaches based on the AHP criticism published in the literature. Particularly, consistency indices proposed originally by T. Saaty and later on by W. Koczkodaj will be discussed and some deficiency of the first one will be demonstrated. Moreover, the priority vector derived by eigenvalue method (EVM) will be investigated in comparison with the geometric average method (GAM). The most important advantages and disadvantages of both approaches will be dealt with in Section 4. In Section 5, new revisited algorithm of AHP method is proposed and new software tool is mentioned. Section 6 is the conclusion part.

2. THE ELEMENTS OF THE PC MATRIX AND AHP

Generally, a DM problem can be characterized by the set of n alternatives, $X = \{x_1,x_2,...,x_n\}$ (objects, persons, DM criteria) which should be ranked from the best to the worst, or vice-versa, based on information given in the pairwise comparisons matrix, $A =\{a_{ij}\}$. A crucial step in a DM process is the determination of a weighted ranking, i.e. rating, on a set X of alternatives with respect to criteria or experts. A way to determine the rating is to start from a relation represented by the PC matrix $A = \{a_{ij}\}$; each element of this matrix a_{ij} is a nonnegative real number which expresses how much x_i is preferred to x_j.

The elements a_{ij} of the PC matrix $A=\{a_{ij}\}$ are taken from a scale S depending on a DM problem, e.g.:

$S = \{0,1\}$ – binary scale,

$S =\{{}^1/_9, {}^1/_8,\dots,{}^1/_2, 1, 2, \dots ,9\}$ – AHP scale,

$S =]0 ; +\infty [$, or $S =]-\infty ;+\infty[$ – interval scale,

$S = [0 ; 1]$ – unit interval scale,

S = an open interval scale equipped with a group operation.

Analytic Hierarchy Process (AHP) is a well-known method for solving decision making (DM) problems of finding the „best“ alternative among the given set of alternatives. This method is frequently used when evaluating, or, more generally, when ranking objects. Pairwise comparisons (PC) method is an intrinsic element of AHP, particularly one of the 3 principles of AHP: hierarchy principle, PC principle, and aggregation principle. In Figure no. 1, as an example, we consider a simple decision problem depicted by 3-level hierarchy. Here, the PC principle is applied on the second level, particularly when evaluating the weights of the criteria, i.e. relative importance of the criteria. Moreover, the criteria could be either quantitative and/or qualitative. Evaluating qualitative criteria, PC method can be used on the third level, see Figure no. 1.

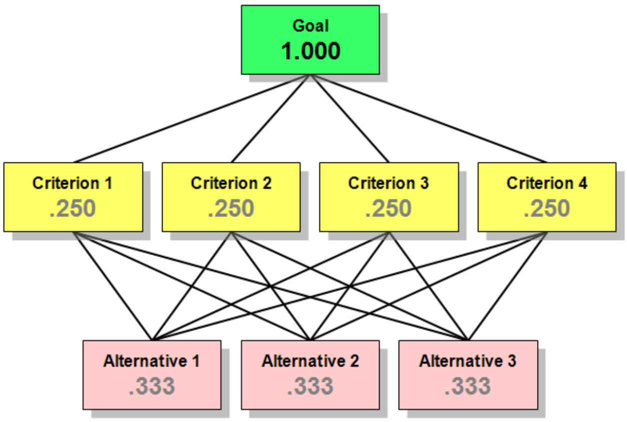

Figure no. 1 – Three-level hierarchical structure of DM problem with 4 criteria and 3 alternatives

3. RECIPROCITY AND CONSISTENCY

Reciprocity and consistency are the most important properties of PC matrix. We say that a PC matrix $A = \{a_{ij}\}$ is *reciprocal*, if:

$$a_{ij}.a_{ji} = 1, \text{ or } a_{ji} = 1/a_{ij} \quad \text{for all} \quad i, j \in \{1,2,\ldots,n\} \tag{1}$$

A PC matrix $A = \{a_{ij}\}$ is *consistent*, if:

$$a_{ik} = a_{ij}.a_{jk}\ , \text{ for all } \ i, j, k \in \{1,2,\ldots,n\} \tag{2}$$

If (2) is not satisfied for some i, j, k, we say that A is *inconsistent*.

The fundamental result concerning consistency is formulated in the following basic theorem.

Basic Theorem (Saaty, 1991):
$A = \{a_{ij}\}$ is consistent whenever there exists a vector $w = (w_1, w_2, \ldots, w_n)$ such that

$$a_{ij} = w_i/w_j \quad \text{for all} \quad i, j \in \{1,2,\ldots,n\} \tag{3}$$

Example 1
Let $A = \{a_{ij}\}$, $B = \{b_{ij}\}$, a_{ij} , $b_{ij} > 0$, be 3×3 PC matrices given as follows:

$$A = \begin{bmatrix} 1 & 2 & 6 \\ \frac{1}{2} & 1 & 3 \\ \frac{1}{6} & \frac{1}{3} & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 4 \\ \frac{1}{2} & 1 & 3 \\ \frac{1}{4} & \frac{1}{3} & 1 \end{bmatrix}.$$

It is easy to check (1), (2) and verify that A, B are reciprocal, A is consistent and B is inconsistent, as $b_{13} \neq b_{12} . b_{23}$, i.e. $4 \neq 2.3 = 6$.

Let $A = \{a_{ij}\}$ be a PC matrix with positive elements. It is natural to assume reciprocity of PC matrix, in real situations PC matrices are, however, always inconsistent. The measure of consistency of the PCM – *consistency index CI* - is a function of matrix elements, such that $CI = 0$, whenever the PC matrix is consistent. Here, we shall introduce and discuss most popular types of consistency indices.

4. CONSISTENCY

In this section we shall deal with two approaches how to measure the consistency of a PC matrix $A = \{a_{ij}\}$ by consistency index. The first consistency index was introduced in (Saaty, 1977) and it is based on the eigenvalue of $A = \{a_{ij}\}$, the second method based directly on the definition of consistency (2) was proposed in (Koczkodaj, 1993).

4.1 Measuring consistency - consistent ratio

Let $A = \{a_{ij}\}$ be an $n \times n$ PC matrix with positive elements, then by Perron-Frobenius theorem, see e.g. Gavalec *et al.* (2014), there exists a unique positive *eigenvalue* of A, λ_{max}, and positive *eigenvector* $w = (w_1, w_2, \dots, w_n)$ such that:

$$Aw = \lambda_{\max} w \quad \text{or} \quad \sum_{j=1}^{n} a_{ij} w_j = \lambda_{\max} w_i \quad i = 1,2,\dots,n \tag{4}$$

Theorem (Saaty, 1991):
$\lambda_{max} = n$ whenever $A = \{a_{ij}\}$ is consistent, otherwise:

$$\lambda_{max} > n \tag{5}$$

T. Saaty defined *consistency index CI* of A as follows:

$$CI(A) = \frac{\lambda_{max} - n}{n-1} \tag{6}$$

and, *consistency ratio* as:

$$CR(A) = \frac{CI(A)}{RI(n)} \tag{7}$$

Here, $RI(n)$ is so called *random index* which is defined as the mean value of CIs for positive reciprocal PC matrices of dimension n. The values of $RI(n)$ for $n = 3,4,\dots,15$ can be found e.g. in (Saaty, 1991). By Saaty´s arguments, the consistency ratio "should be less than 0.1, i.e. $CR < 0.1$. Some PC matrices have $CR < 0.1$, however, their "intuitive consistency" is bad!

Example 2

"Corner" PC matrix:

$$CPC(n,x)=\begin{bmatrix}1 & 1 & \cdots & 1 & x\\ 1 & 1 & 1 & & 1\\ \vdots & \vdots & \cdots & \ddots & \vdots\\ 1 & 1 & \cdots & 1 & 1\\ \frac{1}{x} & 1 & \cdots & 1 & 1\end{bmatrix}.$$

Theorem: $CI(CPC(n,x)) \le \frac{x}{n^2}$.

Evidently, for x = 10, $CI(CPC(10,10)) < 0.1$. Intuitively, $CPC(10,10)$ is not consistent, as element x = 10 is substantially different to other elements (i.e. 1s) of the matrix.

4.2 Measuring consistency - alternative index

Here, we define a different concept of consistency measure which is based on (2), see Koczkodaj (1993). A motivation is based on the equality:

$$a_{ij} = a_{ik}a_{kj} \quad \Leftrightarrow \quad 1-\frac{a_{ik}a_{kj}}{a_{ij}} = 0 \quad \forall i,j,k .$$

Definition: Let A ={a_{ij}} be PC matrix. The (*Koczkodaj's*) *consistency index KI*(A) is defined as:

$$KI(A) = \max_{1\le i,j,k\le n}\left\{1-\min\{\frac{a_{ij}}{a_{ik}a_{kj}},\frac{a_{ik}a_{kj}}{a_{ij}}\}\right\} \tag{8}$$

Notice that A is consistent whenever $CI(A) = CR(A) = KI(A) = 0$, where $CI(A)$ is the consistency index by T. Saaty. Moreover, In contrast to Saaty's consistency index $CI(A)$ that is unbounded in its values, the maximum value of $KI(A)$ = 1. This property enables consistency comparing for various PC matrices with different dimensions n. It is clear that if A is inconsistent, then $CI(A) \neq CR(A) \neq KI(A)$.

By a similar approach to the question of an "acceptable" measure of inconsistency we consider that $KI(A)$ should not exceed the value 0.1 (i.e. 10 % of the range [0 ; 1]).

Example 3

Let B ={b_{ij}}, $b_{ij} > 0$, be a 3×3 PC matrix as follows

$$B=\begin{bmatrix}1 & 2 & 4\\ \frac{1}{2} & 1 & 3\\ \frac{1}{4} & \frac{1}{3} & 1\end{bmatrix}.$$

It can be easily shown that B is inconsistent as $b_{13} \neq b_{12}.b_{23}$, i.e. $4 \neq 2.3 = 6$.

Saaty's consistency index:

$$CI(B) = 0.009, CR(B) = \frac{CI(B)}{RI(3)} = \frac{0.009}{0.5} = 0.017$$

and Koczkodaj's consistency index: $KI(B) = 1 - min\{\frac{4}{6},\frac{6}{4}\} = 0.333$, whereas $CR(B) < 0.1$ and hence the inconsistency of B is acceptable.

On the other hand, $KI(B) > 0.1$ saying that the inconsistency of B is unacceptable.

5. PRIORITY VECTOR

Let $A = \{a_{ij}\}$ be an $n \times n$ PC matrix (i.e. positive, reciprocal square matrix). *Priority vector (PV)* of PC matrix A associated with n alternatives $x_1, x_2, ..., x_n$ is an n-vector $\boldsymbol{w} = (w_1, \ldots, w_n)$ with positive components (calculated from the elements of A) such that w_i denotes the relative importance of x_i, $i=1,2,\ldots,n$, such that: x_i is „not worse than" x_j whenever $w_i \geq w_j$.

Here, we deal with two most popular methods for calculating PV:

1. Saaty´s EVM and
2. GAM (LLSQM).

5.1 Eigenvector method - EVP

Let $A = \{a_{ij}\}$, $a_{ij} > 0$, be a PC matrix, $\boldsymbol{w} = (w_1, \ldots, w_n)$, $w_j > 0$, be a vector satisfying

$$Aw = \lambda_{\max} w \quad \left(\text{or} \quad \sum_{j=1}^{n} a_{ij} w_j = \lambda_{\max} w_i\right) \quad i = 1,2,..., n \tag{9}$$

Normalized solution $\boldsymbol{w}(A) = (w_1, \ldots, w_n)$ of (9) is an eigenvector associated to λ_{max} is called the *priority vector* of A (also called *vector of weights) generated by EVP method*.

Notice, that if A is consistent PC matrix, then by Basic theorem the priority vector $\boldsymbol{w}(A)$ satisfies:

$$A = \{a_{ij}\} = \{w_i / w_j\} \tag{10}$$

Then each normalized row of A is a PV.

If A is inconsistent then the priority vector $\boldsymbol{w}(A)$ can be calculated iteratively by Wielandt's theorem, see Saaty (1991):

$$w(A) = \lim_{k \to \infty} \frac{A^k e}{e^T A^k e}, \quad e = (1,1,...,1) \tag{11}$$

Wielandt's theorem calculates PV not directly but iteratively. The calculation stops if some consecutive iterations are sufficiently close each other.

5.2 Geometric average method - GAM (Logarithmic Least Squares Method)

Let $A = \{a_{ij}\}$, $a_{ij} > 0$, be a PC matrix, Logarithmic Least Squares minimization problem (LLSQ) is the following optimization problem:

$$\text{minimize} \qquad \sum_{i,j=1}^{n} \left(\ln a_{ij} - \ln \frac{u_i}{u_j}\right)^2 \tag{12}$$

$$\text{subject to} \qquad \sum_{i=1}^{n} u_i = 1, \;\; u_i \geq 0 \tag{13}$$

Optimal solution $\boldsymbol{u}(A) = (u_1,\ldots,u_n)$, of (12), (13) can be expressed as the geometric averages of rows of A as follows:

$$u_i = \frac{(\prod_{j=1}^{n} a_{ij})^{\frac{1}{n}}}{\sum_{i=1}^{n}(\prod_{j=1}^{n} a_{ij})^{\frac{1}{n}}},\ i=1,\ldots,n \tag{14}$$

Optimal solution $\boldsymbol{u}(A) = (u_1,\ldots,u_n)$ of (12), (13) is called the *priority vector (PV)* of A (also called *vector of weights) generated by EVP method*.

Notice that a PV generated by GAM satisfies the following:

– If A is consistent PC matrix, then both methods give the same results – same priority vectors, i.e. $\boldsymbol{w}(A) = \boldsymbol{u}(A)$.

– If $n = dim(A) \le 3$ then both methods give the same results - priority vectors: $\boldsymbol{w}(A) = \boldsymbol{u}(A)$.

– If A is inconsistent PC matrix and $n = dim(A) > 3$, then both methods may yield the different results, see Saaty and Vargas (1984).

– EVM violates *independence-of-scale-inversion condition (IOSIC)*, whereas GAM satisfies IOSIC condition. IOSI condition means that if you change e.g. a minimizing criterion (*price*) for maximizing by inversion (i. e. $\frac{1}{price}$), then the final rank of alternatives will not change. This property is important as the aggregation of criteria by weighted average is allowed only for the criteria of the same type maximizing criteria (i.e. profit type criteria). For more details, see Barzilai (1997, 1998).

– Rank preservation condition (RPC) is understood as the following property of the rank generation method:

If any of the existing alternatives within the given set of alternatives is removed and the rank generation method is applied, then the relative ranking of the remaining alternatives does not change. On the other hand, if a new alternative is added to the group of existing alternatives and then the rank generation method is applied, then the relative ranking of the old alternatives does not change.

Rank-preservation condition is violated by EVM, however, satisfied by GAM, see Bana e Costa and Vansnick (2008), Saaty *et al.* (2009).

– In their attempt to disqualify synthesis with the geometric average in Saaty *et al.* (2009), the authors states that "...synthesizing priorities derived in any manner by raising them to the power of the priority of the corresponding criterion and then multiplying them has the shortcoming that $0 < x < y < 1$ and $0 < p < q$ implies $x^p > y^q$ for some p and q. This means that an alternative that has a smaller value under a less important criterion is considered to be more important than an alternative that has a larger value under a more important criterion, which is absurd..." This statement is true, it has, however, nothing in common with GAM method. Here, the above mentioned statement is not true, as for $p = q$, which is the case of GAM, we obtain $x^p < y^p$.

Example 4

Let $A = \{a_{ij}\}$ be a 5×5 PC matrix:

$$A=\begin{bmatrix}1 & \frac{1}{4} & \frac{3}{4} & \frac{1}{2} & \frac{3}{2}\\ 4 & 1 & 3 & 2 & 6\\ \frac{4}{3} & \frac{1}{3} & 1 & \frac{2}{3} & 2\\ 2 & \frac{1}{2} & \frac{3}{2} & 1 & 3\\ \frac{2}{3} & \frac{1}{6} & \frac{1}{2} & \frac{1}{3} & 1\end{bmatrix}.$$

It can be verified that A is consistent, hence the priority vectors generated by EVM and GAM are the same: $\boldsymbol{w}(A) = \boldsymbol{u}(A) = (0.111; 0.444;0.148; 0.222; 0.074)$. Therefore, the rank of alternatives is as follows: $rank(x_1) = 4$, $rank(x_2) = 1$, $rank(x_3) = 3$, $rank(x_4) = 2$, $rank(x_5) = 5$.

Example 5

Let $B = \{b_{ij}\}$ be a 5×5 PC matrix, see (Saaty and Vargas, 1984):

$$B=\begin{bmatrix}1 & \frac{1}{6} & \frac{1}{3} & \frac{1}{8} & 5\\ 6 & 1 & 2 & 1 & 8\\ 3 & \frac{1}{2} & 1 & \frac{1}{2} & 5\\ 8 & 1 & 2 & 1 & 5\\ \frac{1}{5} & \frac{1}{8} & \frac{1}{5} & \frac{1}{5} & 1\end{bmatrix}.$$

It can be verified that B is inconsistent, and the priority vectors generated by EVM and GAM are different: $\boldsymbol{w}(B) = (0.081; 0.346;0.180; 0.355; 0.038)$, $\boldsymbol{u}(B) = (0.073; 0.358; 0.187; 0.345; 0.036)$. The rank of alternatives given by EVM is as follows:

$$rank(x_1) = 4,\ rank(x_2) = 2,\ rank(x_3) = 3,\ rank(x_4) = 1,\ rank(x_5) = 5.$$

The rank of alternatives given by GAM is as follows:

$$rank(x_1) = 4,\ rank(x_2) = 1,\ rank(x_3) = 3,\ rank(x_4) = 2,\ rank(x_5) = 5.$$

As we can see, using the GAM instead of EVM, the alternatives x_2 and x_4 have interchanged the rank: by EVM, the best alternative is x_4, the second best is x_2, however, by GAM, the best alternative is x_2, the second best is x_4.

Now, let us summarize consequences in the form of AHP revisited method. Here, we consider intangible (qualitative) criteria.

5.3 PC method (AHP) revisited – Algorithm and software

Based on the criticism of the original AHP method described in the previous section we propose a revisited algorithm with the following steps:

Step 1. Make a hierarchical structure of the problem (Goal, Criteria, Alternatives).

Step 2. Evaluate weights of criteria by PC method by using GAM (do not use EVM).

Step 3. Evaluate intangible (qualitative) criteria by PC method using GAM (do not use EVM).

Step 4. Calculate consistency index KI (do not use CI).

Step 5. If KI is not sufficiently small, repair the evaluations.

Step 6. Aggregate the results by the usual way.

Having in mind the above mentioned steps of the revisited AHP method we developed a software tool named DAME (Decision Aid Module in Excel), see Ramik and Perzina (2015). This new Microsoft Excel add-in DAME is completely free and was developed to support users in multi-criteria decision making situations. It can be also used by students to help them understand the basic principles of multi-criteria decision making, as it doesn't behave as a black box, however, it can display also results of all intermediate calculations. The proposed software package is demonstrated on a number of illustrating examples.

6. CONCLUSION

In this paper, we have dealt with two essential elements of AHP: measuring consistency of PC matrix and eliciting a priority vector by which the final ranking of alternatives is settled. Classical approaches introduced by T. Saaty in AHP have been compared with later approaches based on the AHP criticism published in the literature. Particularly, consistency indices proposed originally by T. Saaty and later on by W. Koczkodaj have been discussed and some deficiency of the first one has been demonstrated. Moreover, the priority vector derived by eigenvalue method (EVM) has been investigated in comparison with the geometric average method (GAM). The most important advantages and disadvantages of both approaches have been highlighted and discussed. A new revisited algorithm of AHP method has been proposed and new software tool has been mentioned.

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