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Modelling Environment Changes for Pricing Weather Derivatives

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Abstract

This paper focuses on modelling environment changes in a way that allows to price weather derivatives in a flexible and efficient way. Applications and importance of climate and weather contracts extends beyond financial markets and hedging as they can be used as complementary tools for risk assessment. In addition, option-based approach toward resource management can offer very special insights on rare-events and allow to reuse derivative pricing methods to improve natural resources management. To demonstrate this general concept, we use Monte Carlo and stochastic modelling of temperatures to evaluate weather options. Research results are accompanied by R and Python code.

Keywords: weather derivatives; temperature modelling; Monte Carlo.

JEL classification: G13; G17; G18.

1. INTRODUCTION

Environment changes affect businesses and individuals alike, with long-term consequences being very hard to assess and quantify. While there is still a debate going on about the significance of the global environmental changes there are areas like energy sector, utilities and agriculture to name a few, where question is already not “if there are changes” but “how to measure and manage them”. In this paper we put emphasis on weather derivatives as a tool giving a chance to bind together complex environmental models, the impact on the economy from environmental changes (in terms of financial losses and gains) and needed government policies and regulations. Applications and importance of climate and weather contracts extends beyond financial markets and hedging as they can be used as complementary tools for risk assessment. In addition, option-based approach toward resource management can offer very special insights on rare-events and allow to reuse option pricing methods to improve natural resources management. Out of all possible derivative types we focus on options as their features naturally match the decision making process and option portfolios can be used to successfully model interactions between economic agents.

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The derivative based approach is useful as it allows to model asymmetric results/ outputs and value the intrinsic flexibility that economic agents have embedded in their own decisions. Weather derivatives have been applied in risk assessment in Turvey (2001) and Cao and Wei (2004) as well as a tool facilitating insurance in agriculture (Vedenov and Barnett, 2004; Kunreuther and Lyster, 2016; Hofmann and Pooser, 2017). The impact of weather derivatives can be much broader as pointed out in Purnanandam and Weagley (2016) and increase the scrutiny of government agencies operation. We argue that the positive effects from the weather derivatives spread far beyond the improvement of measurement accuracy, since their valuation can be used to steer regulations and assess the efficiency of government policies. In this context it is also not absolutely necessary to have an actively traded contract type in order to benefit from its impact – as long as derivative characteristics and logic is taken into account in the decision making process the quality accuracy of the regulations can be improved.

Yet the use of weather derivatives is not problem free and regarding the goals of this study, problems can be split into two categories:

- Common problems that exist regardless of the contract type and the underlying asset specifics.

Common issues are often related to the "model risk" which arises from selecting a model that is not relevant to the problem being studied. Cont (2006) has suggested a quantitative framework for measuring the model uncertainty, yet for this paper we are mainly interested in model properties with regard to:

- Its capability to match the properties of the analysed data, which in the specific case of weather derivatives refers to being able to explain changes in the market/trade value of the derivatives as well as to agree with observed behaviour of the underlying asset value.
- Its efficient calibration based on the available historical information, which in the specific case of weather derivatives refers to being able to calibrate the model based on limited number of price quotes and also with non-tradable underlying asset.
- Its use for resolving cases of practical importance, which in the specific case of weather derivatives refers to being able to value different contract types.

A number of empirical studies demonstrate that no single model can offer superior performance for all possible types of derivative contracts. Therefore different model selection frameworks have been proposed (Cai *et al.*, 2015; Shcherbakov and Larsson, 2016; Orbay *et al.*, 2016) but in this paper we shall stick to a model selection that is based on the characteristics of the underlying asset and is not driven by an optimization procedure. The selection frameworks described above can still be applied to weather derivatives in combination with our approach, where valuation method is chosen by the researcher and then model dimensions and assumptions are selected through a predefined procedure.

- Problems that only occur depending on the underlying asset type or the contract conditions.

Weather derivatives and weather options are classified as "exotic derivatives", which indicates that either their underlying asset is a very specific one, their payoff is calculated over a complex formulae and/or they are developed for a limited number of customers (or in the extreme case for a specific customer). That complicates the valuation process, since the following conditions are met:

- Due to having a special underlying asset (like for example temperature or rainfall levels) we cannot directly use the "no-arbitrage" approaches in valuation. Lack of tradable asset limits the number of valuation methods usable for weather derivatives.

Lack of tradable asset complicates the valuation process, because it is no longer possible to assume setting up a self-financing trade strategies in the underlying asset that actually replicate the payoff of the analyzed derivative instrument. That automatically rules out use of many existing models that assume tradability, since their application would provide results which are not consistent with the actual underlying asset.

- Weather derivatives payoffs are often calculated in a more complex way than in vanilla derivatives (like vanilla options). For example the first officially recognized weather derivative was an option embedded into an electric energy purchase contract, which stipulated price rebates in case of temperature going below the expected value (Considine, 2000) – thus behaving like a barrier option.

Weather derivatives traded on CME have similar way of calculating the payoffs based on temperature indexes using either deviation from a predefined value of 18 degrees Celsius (thus accounting for heat degree days (HDD) or cooling degree days (CDD)) or accumulating the daily average temperatures ("Temperature Based Indexes", 2017).

Taking into account both common and specific issues related to valuation of weather derivatives we can conclude that it is necessary to use an approach that is both generic enough to integrate complex payoff valuation and at the same time tuned for option valuation, as weather options are the primary instrument of interest.

2. THE MODEL

Lack of tradable underlying asset and complex payoffs dictate that standard option pricing models are not immediately applicable for valuation of weather options. Yet the importance of these derivatives makes it necessary to be able to value them with sufficient accuracy. As a result, the following general approaches have been used in practice:

- Valuation based on historical data analysis.

This is perhaps the simplest (thus least accurate) approach which relies on projecting historical payoffs and value of the derivative in order to estimate future value. As this approach does not take into account changes in the environment or market conditions over time it is expected that the obtained results will be less accurate.

- Valuation based on expected damage or profit.

This approach is based on the idea that weather derivatives can be used to hedge against certain risks, thus their value can be derived from assessing the expected damage (resp. profit) and then treating derivative contract price as insurance against those risks. This approach may be hard to implement in practice as it requires to estimate first the expected damages/profits and then use these values in calculating the option value.

- Valuation based on physical models of the environment.

This approach is based on creating a physical model of the environment and then use forecasts generated by this model in order to estimate weather option value. A specific issue that hinders this approach is the increase of the forecasting error as the forecast horizon increases. Combining different models and scaling up/down existing physical models have been used in order overcome the difficulties of long-term forecasts, however there are still important limitations as for example those discussed in Chun-Fung Lo *et al.* (2008).

- Combined approach of statistical analysis and environment modelling.

The combined approach of statistical analysis and environment modelling aims at using advantages of physical models with the power of extracting long term characteristics of the analyzed processes. In this way the forecasting horizon can be expanded with statistical model supporting the analysis and helping limit the forecasting error. Stochastic modelling has a wide range of financial applications (Ivanov *et al.*, 2013) and can be used to simulate future values of the analyzed process.

In this paper we use a combination of temperature modelling with Monte Carlo simulation in order to value an Asian-like temperature option. We assume that option payoff is proportional to difference between the average temperature (*avg(Temp)*) over the maturity period and the predefined fixed strike value (*E*). Thus we shall conduct our analysis using this difference and assume the notional amount (*M*) used to multiply the temperature difference and calculate the money payoff is equal to 1:

$$Payoff = M.\max\{avg(Temp) - E, 0\} = \max\{avg(Temp) - E, 0\}$$

$$C = e^{-rT}.M.\max\{avg(Temp) - E, 0\} = e^{-rT}.\max\{avg(Temp) - E, 0\} \quad (1)$$

$$P = e^{-rT}.M.\max\{-(avg(Temp) - E), 0\} = e^{-rT}.\max\{-(avg(Temp) - E), 0\}$$

where equation (1) can be adjusted easily for Asian options that use geometric averages, instead of arithmetic ones.

Table no. 1 shows two processes that we have selected to model temperature changes. The first one is a standard mean-reverting process (in our case Ornstein-Uhlenbeck), while the second one assumes there could be jumps in the temperature (due to sudden changes in the environment conditions).

Table no. 1 – Underlying asset model summary

Temperature model A	**Temperature model B**
Temperature can be described with Ornstein-Uhlenbeck (mean-reverting) process described by (2).	Temperature can be described with Ornstein-Uhlenbeck (mean-reverting) process with seasonality and jump diffusion described by (3).
(1) $dTemp_t = \theta(\mu - Temp_t)dt + \sigma dW_t$	(3) $Temp_t = f(t) + S_t$ $f(t) = p_1 \sin(2\pi t) + p_2 \cos(2\pi t) + p_3 \sin(4\pi t) + p_4 \cos(4\pi t) + p_5$ $dS_t = \theta(\mu - S_t)dt + \sigma dW_t + J(\mu_j, \sigma_j)d\Pi(\alpha)$
Mean reversion speed (θ) and mean (μ) are constant values.	$f(t)$ represents the seasonal part of the model, while S_t is the mean-reverting process with jumps that are driven by a Poisson process with density α. Instead of selecting the seasonal component separately we have used the results from electricity prices research from Seifert and Uhrig-Homburg (2007).
Temperatures are modelled and estimated on a daily basis.	Temperatures are modelled and estimated on a daily basis with a single jump (e.g. in case there are several jumps within the same day, we consider them as only one jump in the temperature).
When calculating option prices, we assume that risk-free interest rates and risk-premiums remain constant. Estimation of varying risk-premiums can be added using the methodology of Engle *et al.* (1987).	

With regard to jump diffusion process (model B) we assume that jumps are normally distributed with their own mean (μ_j) and standard deviation (σ_j). These parameters are also estimated during model calibration with historical temperature data.

3. NUMERICAL RESULTS

To calibrate the models, we have used publicly available temperature data covering the period of 01/01/2000 to 01/01/2017. There are multiple sources and weather stations offering information on temperature for this period, but we have decided to use information from airport locations for Plovdiv and Iasi, which is available from Weather Underground website (https://www.wunderground.com/) – parsing and model code available at https://github.com/drnmy/wunderground_parse. Table no. 2 shows the most important properties of the input data. The reason for the much smaller number of observations available for Iasi is that Weather Underground data has huge gaps for 2001, 2002, 2003, 2007, 2008, 2009 and very few observations for 2000, 2004, 2005 and 2006. Yet we have decided to stick to these inputs as they will also be able to demonstrate the model use in cases when there is less data available.

Table no. 2 – Input data summary

Temperature data Plovdiv (ICAO: LBPD)	Temperature date Iasi (ICAO: LRIA)
Data period: 01/01/2000 – 01/01/2017	Data period: 01/01/2000 – 01/01/2017
Obs. used: 6447 daily averages	Obs. used: 2523 daily averages
Mean value: 12.93175, Std. dev: 8.98	Mean value: 12.88205 , Std. dev: 8.223
Forecast period: 90 days (starting as of 01.01.2017)	Forecast period: 90 days (starting as of 01.01.2017)

In order to calibrate the Ornstein-Uhlenbeck (O-U) parameters we have used maximum likelihood estimates approach implemented in R that minimizes the following log-likelihood function (van den Berg, 2011), under the assumption that time step is equal to 1 day:

$$\hat{\sigma} = \sigma\sqrt{\frac{1-e^{-2\theta\Delta t}}{2\theta}}$$

$$f(Temp_{t+1}|Temp_t\colon \theta,\mu,\hat{\sigma}) = \frac{1}{\sqrt{2\pi\hat{\sigma}^2}}e^{-\frac{((Temp_t - Temp_{t-1}e^{-\theta\Delta t} - \mu\left(1-e^{-\theta\Delta t}\right))^2}{2\hat{\sigma}^2}} \tag{4}$$

$$L(\theta,\mu,\hat{\sigma}) = \sum_{i=1}^{n} \ln(Temp_t Temp_{t-1}\colon \theta,\mu,\sigma)$$

Estimated parameters of the O-U process are then used to simulate the temperature values over a 90 day horizon with Monte Carlo simulation and 10,000 calculated temperature paths. The average value of each path is compared with predefined strike of 18 degrees Celsius in order to estimate an Asian weather option.

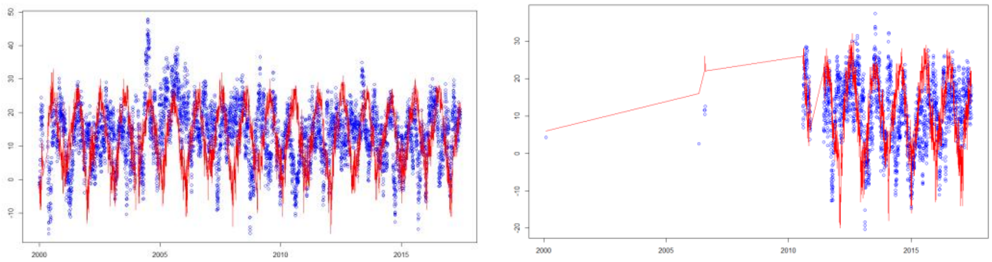

Figure no. 1 – Estimated (red) and actual (blue) values of the O-U temperature modelling for Plovdiv (left) and Iasi (right)

Figure no. 1 shows the estimated values of the temperature using O-U processes for Plovdiv and Iasi, compared to the original observed data, while Table no. 3 contains the parameter estimates and the calculate option value using Monte Carlo simulation with fixed discount rate of 1% and period of 90 days. Results show that there is a slight difference in the mean reversion speed between estimates for Plovdiv and Iasi, however it is not large and significant (which can be proven by bootstrapping). Option valuation shows similar values for call and put options, though the difference is large for the Asian call, which is mainly due to the different estimates in the mean and standard deviation.

Table no. 3 – Estimates of the O-U parameters and respective option valuation

O-U process calibration and option value for Plovdiv	O-U process calibration and option value for Iasi
$dTemp_t = 0.0508(12.9349 - S_t)dt + 2.8619dW_t$	$dTemp_t = 0.0406(12.8317 - S_t)dt + 2.3653dW_t$
Estimated price (based on temp difference) for: 90 day arithmetic Asian call and E=18 C: 0.1019866 90 day arithmetic Asian put and E=18 C: 7.990141	Estimated price (based on temp difference) for: 90 day arithmetic Asian call and E=18 C: 0.0677214 90 day arithmetic Asian put and E=18 C: 8.381596

Table no. 4 presents the process calibration results (of the stochastic part of the model) and option valuation for the second temperature model. As expected when taking into account seasonal component, the mean reversion speed is higher in both studied locations and the standard deviation is lower, compared to the plain Ornstein-Uhlenbeck model. Seasonal adjustments also allow to clarify in more details weather specific of both studied locations, as we can see in the jump component estimates. Although in both cases we assume that jumps are normally distributed, mean and standard deviation for Iasi case have larger absolute values indicating that expected sudden changes in the temperature are larger there.

Table no. 4 – Estimates of the O-U parameters with seasonality and jumps and respective option valuation

O-U process with seasonality and jumps calibration and option value for Plovdiv	O-U process with seasonality and jumps calibration and option value for Iasi
$dS_t = 0.7255(-0.0437 - S_t)dt + 1.6645dW_t + J(0.0964,2.4333)d\Pi(0.3247)$	$dS_t = 0.8153(0.3258 - S_t)dt + 1.7545dW_t + J(-0.8294,2.8128)d\Pi(0.321$
Estimated price (based on temp difference) for: 90 day arithmetic Asian call and E=18 C: 0.000545 90 day arithmetic Asian put and E=18 C: 8.6523	Estimated price (based on temp difference) for: 90 day arithmetic Asian call and E=18 C: 0.000348 90 day arithmetic Asian put and E=18 C: 9.3928

With regard to option valuation the second model provides better estimates, especially if we consider that the period being examined fits in the winter (beginning of the year) at both locations. Therefore the value of the 90 days call option (benefitting from an increase of the average temperature above the specified threshold) is expected to be low and close to zero, while the put value is expected to be higher (since it is more likely to have lower temperatures in the winter).

4. CONCLUSIONS

Option-based approach toward resource management can offer very special insights on rare-events and allow to reuse derivative pricing methods to improve natural resources management. To demonstrate this general concept, we have used stochastic modelling with mean-reverting processes and Monte Carlo temperatures to evaluate Asian weather options. Using publicly available temperature data we have been able to calibrate two different models – one with standard Ornstein-Uhlenbeck process and a second one including jump-diffusion and forecast different temperature paths over a 90 days horizon, starting from 01/01/2017. Although both models are able to cope with temperature forecasting and they both yield meaningful results for valuated sample Asian weather options, the second one, which includes jumps and accounts for seasonal effects is more accurate.

To sum up, the advantages of using environment modelling go beyond the pure valuation of derivative instruments. Their major advantage is providing a common framework that can fit together stochastic models, management decisions, financial impact (monetized effects) and the effects on individual behavior. That allows to assess not only derivative contracts, but also to forecast and measure the result of regulations and environment policies. Due to the embedded flexibility and asymmetric outputs that are part of the option contracts, the valuation process is useful also when analyzing economic agents and the way they react to environment changes. It is possible to further improve the accuracy of the models and relax some of the assumptions, by including in the model a provision for time-varying risk-premium that could be calculated either from existing futures contracts or by building a separate forecast with econometric tools.

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