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STOCK PRICE SIMULATION USING BOOTSTRAP AND MONTE CARLO

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Abstract

In this paper, an attempt is made to assessment and comparison of bootstrap experiment and Monte Carlo experiment for stock price simulation. Since the stock price evolution in the future is extremely important for the investors, there is the attempt to find the best method how to determine the future stock price of BNP Paribas´ bank. The aim of the paper is define the value of the European and Asian option on BNP Paribas´ stock at the maturity date. There are employed four different methods for the simulation. First method is bootstrap experiment with homoscedastic error term, second method is blocked bootstrap experiment with heteroscedastic error term, third method is Monte Carlo simulation with heteroscedastic error term and the last method is Monte Carlo simulation with homoscedastic error term. In the last method there is necessary to model the volatility using econometric GARCH model. The main purpose of the paper is to compare the mentioned methods and select the most reliable. The difference between classical European option and exotic Asian option based on the experiment results is the next aim of tis paper.

Keywords: European option, Asian Option, bootstrap, Monte Carlo, stock price simulation, modeling volatility

JEL classification: C15, C51, C52, G11

1. INTRODUCTION

The answer on the question, what will be the stock price in the specific time in the future is worth a fortune. Investors around the globe are seeking to know what the evolution of their stocks is in the future. Their motivation is to gain the maxim profit under the conditions of acceptable risk exposure and in the acceptable time range. These three basic attributes of the investments – return, risk and time – are three basic components of the investment triangle. Based on these attributes all investors make their decisions about strategic asset allocation. The requirements vary from investor to investor. The crucial decision factor is the risk aversion of the investor. Some investors are willing to accept higher level of risk, which is compensated with the higher return. On the other side, many investors are satisfied with lower gain, but more certain. The majority of the financial instruments follow random walk, so it is very difficult to predict the direction of their

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movements. In spite of that, there are some patterns, which can be generalized for some asset classes. This patterns help investor in their decision making processes.

Team of researchers Andersen, Bollerslev, Diebold and Varga published in 2006 the results of their extended research about response of US, German and British stock, bond and foreign exchange markets to real-time US macroeconomic news. Their summary statistics confirmed the usual rank ordering in terms of volatility, with stock market being the most volatile, followed by foreign exchange rates, and then fixed income. The only exception to this rule is the US T-Bond market, for which the unconditional return standard deviation exceeds the standard deviations for the three exchange rates. Thus, they found out that T-Bond markets react most strongly to macroeconomic news (Andersen *et al.*, 2006). The conclusions of their research suggest that there is the relationship between the macroeconomic news and development on financial markets. The research published by the Nnorges Bank Investment Management (NBIM) in 2012 concludes that the growth prospects are better in emerging markets than in developed countries for some decades to come due to favourable demographics and healthier public finance. The argument follows the logic that emerging economies have stronger potential growth than established OECD countries based on faster population growth and catch-up productivity (Nnorges Bank Investment Management, 2012a). There are authors who oppose these opinions and emphasize the importance of considering the potentially detrimental influence of the state in emerging countries´ capital markets, the risk of policy mistakes and the likelihood of speculative bubbles and subsequent financial crisis (Smith and Beceren, 2011; Proksová and Bohdalová, 2015). On the other side, the developing countries were generally less vulnerable to the bursting of the real estate bubble in the US and the ensuring financial contagion than many developed countries (Nnorges Bank Investment Management, 2012a). Campbell and Diebold documented in their paper about Stock Returns and Expected Business Conditions in 2005 that there are two different categories of predictors of economic stock returns and expected business conditions. First category is represented by traditional financial predictors like dividend yield, default premia a term premia. The second category is represented by macroeconomic predictors like consumption wealth ratio forecast of future real GDP growth (Campbell and Diebold, 2005). The consumption wealth ratio was developed by Lettau and Ludvigson, in 2001, as a macroeconomic stock return predictor (Lettau and Ludvigson, 2001). Different point of view about investment decision making process offers a paper about Time-varying Expected Returns and Investor Heterogeneity published in March 2012 in NBIM. Based on this paper, the time-varying expected returns and investor heterogeneity are the foundations for rebalancing. Following the investor heterogeneity principle there are two frameworks as standard for accounting volatility of financial markets and predictability of excess return. First framework is called habit specification, which asserts that the investors are more risk averse in recessions, when their consumption is low. They are less risk averse in expansions, when their consumption is high. Second framework is so-called recursive risk – preference specification. This framework concludes that the investors put relatively more weight on the changes in uncertainty associated with long-run growth than short-run consumption fluctuations (Nnorges Bank Investment Management, 2012b).

As was shown, the prediction of the future movement in financial instrument prices is achievable, in spite of the fact, that the prices of majority financial instrument follow random walk. The assets classes are vulnerable to macroeconomic news, also the geographic location can play the role and the precise moment in the business cycle is also very important. From these reasons the valuation methods are adequate for determining the value of the investment instrument. The aim of this paper is to model the stock prices in the following 20 days based on historical data on stock prices the key universal bank BNP Paribas´ based in Paris. The employed simulations models are Monte Carlo simulation and bootstrap method. The comparison of these two methods is the crucial element of the empirical part.

2. STOCK PRICE SIMULATION

The investors seek to know the future price of their investment and the risk associated with this investment. Their motivation is understandable since they demand the certain return of their investment (Bohdalová and Greguš, 2012). Strategic asset allocation has a strong potential to diversify risk and ensure higher return than investing without strategic assets allocation and portfolio rebalancing (Wallick *et al.*, 2012). There is the assumption that successful investor should keep his portfolio well diversified among different asset classes, geographical regions, industries, length of maturities and many other factors. Well diversified portfolio can offset the loss from one particular investment instrument by another instrument. In spite of that, the investors usually seek to predict the possible asset price evolution in order to reduce uncertainty connected with their investment. Therefore, the many different simulation models are employed with one specific target, which is focused on precise asset valuation (Bohdalová and Greguš, 2011).

One of the best known equity can be considered stock. Stock is the basic asset class, which is very often used as an underlying asset for more complicated financial derivatives. The advantage of stocks is the fact, that they are well known and easily understandable. Therefore it is relatively easy to design a model, which would simulate the future prices. Figure no. 1 suggests that the derivative markets are much more attractive for the investors. The amounts traded since 2007 until 2015 on OTC (over-the-counter) markets are extremely high. The attraction of financial derivatives stems from their rich variability and possibility to generate higher returns, hedge the portfolios and speculate on the future conditions of the particular underlying asset.

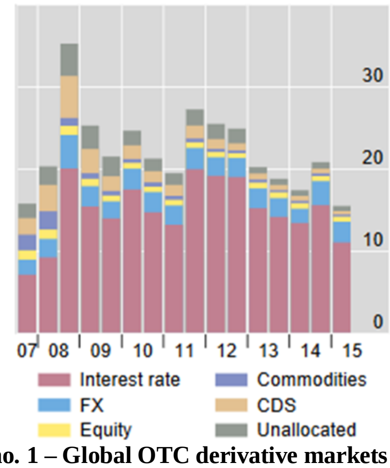

Figure no. 1 – Global OTC derivative markets (USD trn)
Source: Bank for International Settlements (2015)

One of the most traded financial derivatives are *options*. It is well known that the financial options represent the right for its holder to buy or to sell the specific units of the underlying asset at the specific time (maturity date) for a specific amount (exercise price). The flexibility of the options is thus much higher than the flexibility of the simple stock, while the stock can be used as the underlying asset for the option. The option flexibility is even higher due the wide palette of many different options. The basic options categories are European option and American option. The European is the easiest for valuation. It allows the holder to exercise (or realize) the option only on one specific day – maturity date. The market price at maturity date is the key decision factor whether the investor wants to exercise the option or not. In case of American option, the investors have much higher possibilities in terms of realization. The advantage is that investor can exercise the option whenever it is convenient for him. These two option categories are relatively easy for valuation. Slightly more complex methods must be employed in valuation of exotic options. An example of an exotic option can be Asian option. Asian options are options where the payoff depends on the arithmetic average of the price of the underlying asset during the life of the option (Hull, 2012; Franke *et al.*, 2008). The valuation is a bit more complicated than in previous mentioned options.

Once the investor decides which financial instrument should be simulated, there must be done the decision which method should be employed in order to value the instrument as precise as possible. Assume that the investor has decided to invest in the stock of one particular company. The fundamental analysis should precede the cash investment. There are many different valuation methods. As the basic valuation method, can be employed DDM (dividend discount model) forward looking multi-stage model. Dividend discount model summarized by Daly, Nielsen and Oppenheimer in 2010 contains the following four phase (Daly *et al.*, 2010):

- Phase I (years 1 – 2): the assumption of short-run earnings growth forecasts based on a top-down earnings model;
- Phase II (years 3 – 4): the assumption that ROE (return on equity) fades to trend by the end of year 4, and that the earnings change to achieve this occurs equally over the two years;
- Phase III (years 5 – 20): the assumption that profits grow in line with trend real GDP growth and that the pay-out ratio equals the average over the last five years;
- Phase IV (terminal value): in the very long run, the assumption is that ROE is equal to the cost of equity and that profits growth in line with trend GDP growth.

The pitfalls of this approach stem from the plenty assumptions in the process. In the reality it is very difficult to forecast the sales, cash flow, economic growth, ROE and many other factors. Investor usually does not have capacity to come to the reasonable assumptions and forecast would be not confident. Many of the information are internal and company would not be willing share this sensitive information. This method can be applied by the financial department of the company and then approach the investor with the results from the internal valuation.

Different approach to determining the future development is price simulation. Assume the basic stock as the underlying asset for the option. The possible methods how to simulate the evolution of the stock price in the future stem from the historical stock prices of the same stock. The purpose of these methods is to design the model, which would generate the future stock prices using the historical set of data. The simulation is based on the assumption, that the data follow predetermined patterns and the price tomorrow depends on the price observed the day before. The probability to change direction rapidly or to move dramatically is very low. Of course, this may happen, and in the reality the situations like this occur quite often. The triggers of such unexpected changes in the stock price evolution are called shocks and it is almost impossible predict this kinds of shocks. There are some indications like for example OPEC meetings or US elections or new unexpected event, but the final impact is usually very difficult to predict. Therefore it is convenient depend on the simulation model, which would generate the future data. Each simulation is just the simplified model of reality, which means that the output only rarely corresponds with the reality. Although, there are more simulation models, the general set up of a simulation experiment can be designed:

1. Population specification – to choose the historical data and specify data generating process (for example the assumption about the distribution),
2. Draw a new sample from the population based on historical data,
3. Calculate the statistic of interest (mean, variance,…) and save the values,
4. Repeat steps 2 – 4 many times in order to obtain the most representative results. The number of replications is optimal to impose at least 1.000 times.
5. Evaluate the results.

The step two is the key one in the process. The rest steps are very easily to obtain, but draw a new sample requires to do develop a smart algorithm which would generate the data. The process of developing such algorithm creates the logic of the simulation model. The two very often used alternative simulations methods are:

a) Bootstrap,
b) Monte Carlo simulation.

a) **Bootstrap experiment** generates a sample by resampling observed data many times. The sample is treated as the unknown population from which the sample can be drawn using replacement. The bootstrap method is very convenient when the distribution of the underlying data is not known. Therefore the core advantage of bootstrapping comes from the fact the method allows to generate data without making assumptions about the distribution of underlying data. The pitfall of this method is that the sample distribution may be poor proxy for true distribution. Therefore the bootstrap method may fail to generate samples with the same distribution as the original data (Kenett *et al.*, 2006). The bootstrap experiment can be explained very easily. Assume there is a population of 10 stock prices capturing the last 10 days evolution. Each stock price has the index from 1 to 10. There is the task to generate the stock prices for the next three days using only the historical 10 values. First, what is necessary to calculate is the daily return, which can be calculated using following formula:

$$\mu_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

where μ_t is daily return, P_t is stock price today and P_{t-1} is stock price yesterday.

Once the daily returns for the first 10 days are calculating, it is necessary to generate randomly the index from 1 to 10; which would assign the daily return to the current day based on the historical daily return assigned by the index. Thus, as the stock price yesterday and daily return are known, it is easy to calculate stock price today. Re-arranging the formula it is possible to calculate the stock price today:

$$P_t = P_{t-1}e^{\mu_t} \tag{2}$$

with $\mu_t = \sigma_t\varepsilon_t$ and $\varepsilon_t \sim N(0,1)$.

Using the same procedure, it is possible to calculate the stock price tomorrow and the day after. Bootstrap method is thus an easy procedure how to generate new sample from the historical data. In the reality the population must be rich enough to capture the distribution. Once the experiment is applied many times (ideally at least 1.000 times), the average values represent the statistically powerful new sample.

b) **Monte Carlo experiment** is an alternative method how to generate new sample from historical data. The key difference is that the sample is generated in Monte Carlo simulation by drawing from a hypothesised analytical distribution (Hull, 2012; Bohdalová and Šlahor, 2008). Thus, Monte Carlo experiment is convenient to use in cases when the true distribution of the underlying data is known. The main advantage of this method is then that replicated sample follows the same distributional properties as the original data. In the reality, the distribution is rarely known (Tsay, 2010; Bohdalová, 2006). Employing wrong assumption about the distribution invalidates the experiment. The general formula, which can be employed for the Monte Carlo experiment is following:

$$P_t = P_{t-1}e^{(U+SE*\sigma)} \tag{3}$$

where P_t is stock price today and P_{t-1} is stock price yesterday, U is drift (or constant) SE is standard error and σ is random shock with normal distribution.

Term $SE*\sigma$ is well known Wiener process, which is the indicator of random walk. The constant can be obtained from linear regression and also the standard error. Since σ is randomly generate from normal distribution, there is incorporate the assumption of normal distribution. The assumption about the normality is a crucial part for successful Monte Carlo experiment. Distribution can be tested by Jarque-Bera (JB) test of normality. If the assumption about the distribution was correct, the simulation can be repeated many time and average values can be considered as statistically powerful new sample.

3. DATA AND METHODOLOGY

The examined data represent daily stock prices of BNP Paribas´ bank since January 3rd, 2000 until January 31st, 2017. The data were recorded only on working days, so no during the weekends. In the case that the value was not recorded from the reason that the stock exchange was closed (during the holidays), the missing values were calculated as the arithmetic average of the value before and after the holidays. Thus, the sample consists of 4.457 observations, which is considered as statistically significant sample. As the data source was used publicly available data warehouse – Yahoo Finance. The data are used as historical sample for the stock price simulation in following twenty days. The historical daily returns are created under following data generating process:

$$\beta_0 + \mu_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{4}$$

with $\mu_t = \sigma_t\varepsilon_t$ and $\varepsilon_t \sim N(0,1)$. This operation consumed one observation – one degree of freedom.

At the beginning there is a short description of BNP Paribas´ bank focused on the financial situation of the bank from the stock markets perspective. The information is gathered based on publicly available resource, especially from the official websites of the bank.

The purpose of the simulation experiment is generation the stock prices for the next 20 days. The intention is to examine two separate scenarios – one considering European option on the stock of BNP Paribas´ and the second scenario considering Asian option on the stock of BNP Paribas´. The initial assumptions are following:

- Today: 31 January 2017;
- Stock price at the last date (31 January 2017): 60.43 EUR;
- Time to maturity: one month (28 February 2017 – 20 days);
- Exercise price: 63 EUR (assumed value).

Two different simulation methods are employed:

a) Bootstrap experiment;
b) Monte Carlo simulations.

Each method considers two distinct cases:

1. Homoscedastic error terms: ${\sigma_t}^2 = \alpha_0$
2. Heteroscedastic error terms, which are expressed using GARCH (1,1) generalised autoregressive conditional heteroscedasticity) model. GARCH models are used for modelling the volatility, in cases when the volatility varies over time – thus the series are not homoscedastic, but they are heteroscedastic):

$$ {\sigma_t}^2 = \alpha_0 + \alpha_1 {\mu_{t-1}}^2 + \alpha_2 {\sigma_{t-1}}^2 \tag{5} $$

The GARCH model is created in the statistical software EViews.

Each experiment is replicated 10.000 times and the average value is stored. For the simulations is used programming software Matlab. The final average values are finally used for the calculation of the intrinsic value of the European option and Asian option. The outputs of the experiments are four different simulations for European and Asian option – one using bootstrap method with homoscedastic error term, one using bootstrap method using heteroscedastic error term, one Monte Carlo simulation using homoscedastic error term and one Monte Carlo simulation using heteroscedastic error term. The comparison of different models and select the best model are the crucial part of the work. The hypotheses are:

H1*: Volatility of the stock caries over time – there data series are heteroscedastic;*

H2*: Value of Asian option is smaller on average than European option on the same stock, because the value of the Asian option is calculated as the average of the prices;*

H3*: Heteroscedastic models should generate a bit higher average returns than homoscedastic models, since the prices in homoscedastic are can be perceived as the average values coming from all observations.*

4. BNP PARIBAS´ STOCK PRICE SIMULATION

BNP Paribas´ is French based bank with headquarter in Paris. The bank operates mainly in Europe, but is active also in America, Asian-Pacific area, Africa and Middle East. BNP Paribas´ has affiliates in 75 countries around the globe. The bank is focused on individual clients, small and medium enterprises and international corporations. It is considered to be one of the biggest European banks with relatively strong investment portfolio. Apart from traditional retail banking, the bank is also specialized in private banking, corporate banking, wealth management, asset management, investment services, insurance, brokerage and real estate. BNP is organized in two main sector based on its activities: Retail Banking & Services (RBS) and Corporate & Institutional Banking (CIB). The main of the income comes from Retail Banking & Services. It can be concluded that BNP Paribas´ is well diversified from geographical, clients and activities point of view (BNP Paribas, 2016).

Figure no. 2 depicts the stock price evolution of BNP Paribas´ since the year 2000 until January 2017. From the figure inspection, it is obvious that data does not tend to revert to one stable value. There is neither evidence of the trend nor the structural break. Based on the rough figure assessment it can be concluded that the stock price of BNP Paribas´ follows random walk. The peak was in 2007 when the global financial crisis started and later on, following the similar evolution on the global stock markets, the stock price wiped out rapidly and in between the years 2008 and 2009 the stock price exhibited the global minimum – almost 0 EUR per share. This negative shocked immediately reverted and it seemed that the stock is following bull market, but the stock evolution stabilized and in 2011 the stock again deteriorated rapidly. This decrease was the reaction on debt sovereign crisis which affected whole financial system. Since that the stock price is more or less increasing. The stock price at 31 January 2017 was 60.43 EUR.

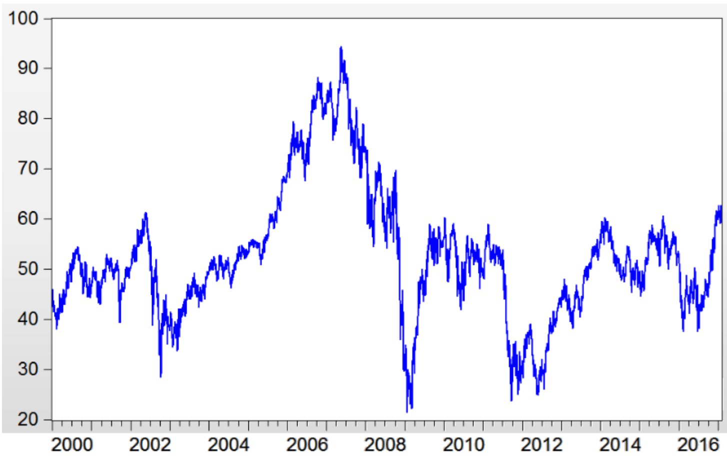

Figure no. 2 – BNP Paribas´ - stock price evolution
Source: own processed based on Financeyahoo (2017)

Figure no. 3 depicts the same evolution, but from the returns perspective. The figure can be interpreted as the track of the volatility. It is obvious that the volatility is not constant over the period. For example around the years 2002 – 2003 the volatility was higher than in later periods 2003 – 2006. The highest volatility was between years 2007 and 2008. Later on the volatility is still not constant. There are some periods, when it seems that the volatility is constant (for example 2003 – 2006 or 2013 – 2015). This pattern, when the big shocks (residuals) tend to be followed by big shocks in either direction, and small shocks tend to follow small shocks, is so-called volatility clustering (Verbeek, 2008). Figure no. 2 and Figure no. 3 provide with the complementary information. While it is possible to observe the equity price on the Figure no. 2, the Figure no. 3 depicts the risk of the same equity. In case the reruns are high, also the risk is high.

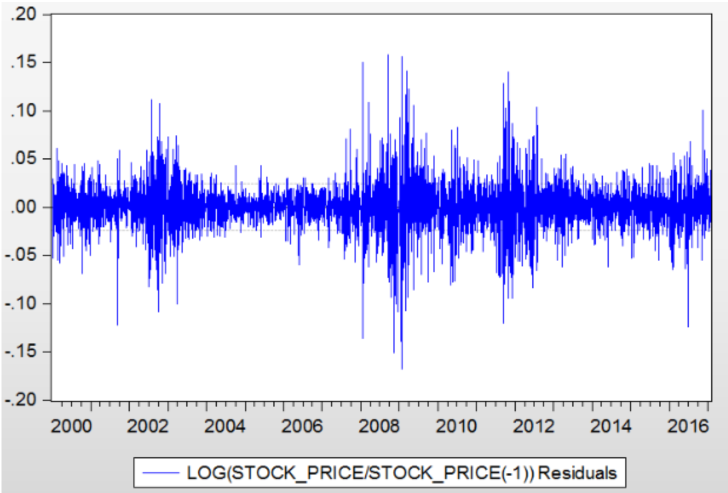

Figure no. 3 – Returns of BNP Paribas´ stock
Source: own processed based on Financeyahoo (2017)

Bootstrap simulation using homoscedastic error term

The bootstrap method is performed based on the following assumption:

- Stock price at 31 January 2017: 60.43 EUR;
- Time to maturity: 20 days;
- Exercise price: 63 EUR.

Bootstrap simulation generates the stock price for next 20 days based on trivial equation:

$$P_t = P_{t-1}e^{\mu_t} \tag{6}$$

with $\mu_t = \sigma_t \varepsilon_t$ and $\varepsilon_t \sim N(0,1)$.

The algorithm replicates the calculation of 20 days 10.000 times and then calculated the average values, which are compared with exercise price 63 EUR separately for European and Asian option. The results are captured in the Table no. 1.

Table no. 1 – Results of Bootstrap experiment with homoscedastic error term

	European option	**Asian option**
Mean	1.7388	0.6895
Variance	12.4943	2.9500
Skewness	2.7667	3.5164
Kurtosis	12.318	18.5559
Jarque – Bera	1.87E+05	5.52E+05
P value	0	0

Source: own processed based on Financeyahoo (2017)

The results confirmed the hypothesis that the European option tends to have higher value than Asian option and the variance is also higher in case of European option. Based on kurtosis, skewness and Jarque - Bera test the null hypothesis about normal distribution can be rejected, thus the distribution is not normal. The shortfall of this method is the fact that the data are not homoscedastic; therefor the output is not reliable. This disadvantage can be reduced in the next model – bootstrap with homoscedastic error term.

Bootstrap simulation using heteroscedastic error term

The bootstrap simulation using heteroscedastic error term stems from exactly the same assumption like the case above. Also the simulation formula is exactly the same. Only difference is the data which are used. In the simulation will not be used the entire sample (4.457 observations). Instead of all sample, there will be used only the last quarter of the total data. It means, only data for the last four years. There are more reasons for this update. Econometric reason stems from the requirement for data homoscedasticity. Indeed, the data in the last four years seems to be homoscedastic, so the volatility seems to be constant. This model is much more convenient are reliable. The economic reason stems from the fact, that the stock price should be generated from the data, which are relevant. There is a decent probability that the evolution 15 years ago is not relevant for the current evolution. The shocks, which occurred very long time ago will hardly affect the data in the next 20 days. The periods with high volatility many years ago are not relevant. Since there is imposed a restriction on the data, the method is called also blocked bootstrap method. The results from the experiment under the changed conditions are following:

Table no. 2 – Results of Bootstrap experiment with heteroscedastic error term

	European option	Asian option
Mean	1.8154	0.7355
Variance	13.4337	3.1844
Skewness	2.7202	3.5117
Kurtosis	11.9758	19.7478
Jarque – Bera	1.72E+05	6.40E+05
P value	0	0

Source: own processed based on Financeyahoo (2017)

European option exhibits again higher value than Asian option and the variance is also higher in case of European option. The null hypothesis about normal distribution can be rejected, which does not invalidate the model, since bootstrap simulation does not require the assumption about the distribution.

Monte Carlo simulation using homoscedastic error term

Employing Monte Carlo experiment, the assumptions will be exactly the same like in the previous cases. The algorithm for generating 20 stock prices is different. The formula, which is the base of the algorithm, is following:

$$P_t = P_{t-1}e^{(U+SE*\sigma)} \tag{7}$$

where P_t is stock price today, P_{t-1} is stock price yesterday, U is drift (or constant) SE is standard error and σ is random shock from normal distribution. The necessary requirement in Monte Carlo simulation is the assumption about the normal distribution. In order to find out the drift and standard error it is necessary to run the linear regression. Output of the linear regression from EViews is shown in Table no. 3.

Table no. 3 – Linear regression for returns

Dependent Variable: LOG(STOCK_PRICE/STOCK_PRICE(-1))
Method: Least Squares
Date: 01/31/17 Time: 17:21
Sample (adjusted): 1/04/2000 1/31/2017
Included observations: 4456 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	6.12E-05	0.000369	0.166153	0.8680

R-squared	0.000000	Mean dependent var	6.12E-05
Adjusted R-squared	0.000000	S.D. dependent var	0.024600
S.E. of regression	0.024600	Akaike info criterion	-4.571918
Sum squared resid	2.695982	Schwarz criterion	-4.570482
Log likelihood	10187.23	Hannan-Quinn criter.	-4.571412
Durbin-Watson stat	2.077237		

Source: own processed based on Financeyahoo (2017)

From the regression, the value of the constant C (or drift) is 6.12E-05 and standard error is 0.0246. Using these parameters in the simulation it is possible to generate stock prices for the next 20 days. The simulation is replicated 10.000 times and the average values are stored and then compared with the exercise price. Thus the characteristics of the European and Asian option can be calculated. The results are in Table no. 4.

Table no. 4 – Results of Monte Carlo experiment with homoscedastic error term

	European option	Asian option
Mean	1.7611	0.7146
Variance	11.7267	2.6727
Skewness	2.4685	2.9184
Kurtosis	9.86828	12.7808
Jarque Bera	9.84E+04	2.08E+05
P value	0	0

Source: own processed based on Financeyahoo (2017)

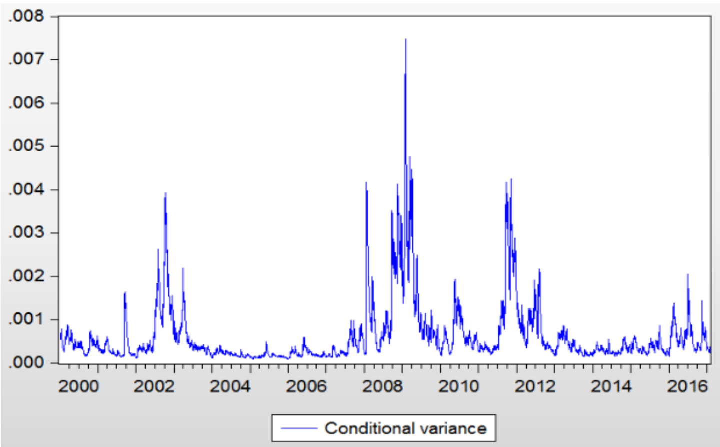

Figure no. 4 – Conditional variance in the residuals
Source: own processed based on Financeyahoo (2017)

Results of the experiment suggest that the European option has also in this case higher value than the Asian option and the same holds for the variance. The skewness, kurtosis and Jarque – Bera test conclude that the series does not have normal distribution, which means that the initial assumption about the distribution was wrong. This finding invalidates the results of the experiment. The possible reason can be heteroscedasticity in the returns, which confirm Figure no. 3 and also Figure no. 4, which depicts conditional variance in the residuals.

Monte Carlo simulation using heteroscedastic error term

The key logic of the last simulation is transfer heteroscedastic returns to homoscedastic series. This can be achieved by ARCH or GARCH model. Firstly, the ARCH model was designed, but the volatility was not captured sufficiently, therefore the GARCH model was imposed. For the most financial time series the simples GARCH(1,1) model is satisfactory (Gujarati and Porter, 2009). Figure no. 5 depicts the standardized residuals after employed GARCH(1,1) model. The figure obviously differs from Figure no. 3.

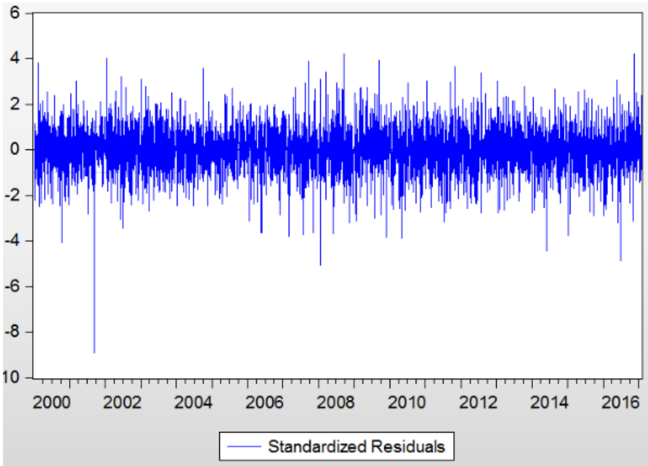

Figure no. 5 – Standardized residuals of GARCH (1,1)
Source: own processed based on Financeyahoo (2017)

The series become homoscedastic employing GARCH (1,1). Now Monte Carlo simulation can be processed. The initial assumptions are the same like in the previous experiments. Monte Carlo algorithm needs to be adjusted due the fact that the data changed after employing GARCH (1,1).

$$P_t = \log(P_{t-1}) + U + SE^*\sigma \tag{8}$$

with:

$$SE^* = \sqrt{a_0 + a_1 \mu_{t-1}{}^2 + a_2 VAR_{t-1}{}^2} \tag{9}$$

where P_t is stock price today; P_{t-1} is stock price yesterday; U is drift (or constant); SE is standard error; σ is random shock from normal distribution, a_0 a_1 a_2 are the parameters from the linear regression μ_{t-1} is the last residual from GARCH robust standard errors model and VAR_{t-1} is the variance of the last residual from GARCH robust standard errors model.

In order to process Monte Carlo simulation, it is needed to run the regression. Since there was imposed GARCH model, the model with robust standard error must be designed. The model is depicted in Table no. 5.

Table no. 5 – Model with robust standard errors

Dependent Variable: LOG(STOCK_PRICE/STOCK_PRICE(-1))
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 01/31/17 Time: 17:39
Sample (adjusted): 1/04/2000 1/31/2017
Included observations: 4456 after adjustments
Convergence achieved after 25 iterations
Coefficient covariance computed using Bollerslev-Wooldridge QML
sandwich with expected Hessian
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000580	0.000263	2.208507	0.0272
Variance Equation				
C	5.99E-06	1.89E-06	3.164305	0.0016
RESID(-1)^2	0.091206	0.012473	7.312109	0.0000
GARCH(-1)	0.900552	0.012840	70.13728	0.0000

R-squared	-0.000445	Mean dependent var	6.12E-05
Adjusted R-squared	-0.000445	S.D. dependent var	0.024600
S.E. of regression	0.024605	Akaike info criterion	-4.943847
Sum squared resid	2.697180	Schwarz criterion	-4.938100
Log likelihood	11018.89	Hannan-Quinn criter.	-4.941821
Durbin-Watson stat	2.076313		

Source: own processed based on Financeyahoo (2017)

From this model, the required parameters are:

$U = 0.00058$ $\quad a_0 = 5.99\text{E-}06$ $\quad a_1 = 0.091206$

$a_2 = 0.900552$ $\quad \mu_{t-1} = -0.01046$ $\quad VAR_{t-1} = 0.000345$

After including these parameters into the algorithm in Matlab, storing the values and replicating 10.000 times, the following results are obtained:

Table no. 6 – Results of Monte Carlo experiment with heteroscedastic error term

	European option	Asian option
Mean	1.8332	0.7431
Variance	12.2943	2.8456
Skewness	2.4110	2.9437
Kurtosis	9.4946	13.1536
Jarque Bera	8.76E+04	2.25E+05
P value	0	0

Source: own processed based on Financeyahoo (2017)

European option reached out higher value than Asian option and also the variance is higher than for Asian option. Test of normality shows that the series does not follow normal distribution, which confirms also the histogram of standardized residual series. The wrong assumption about the distribution invalidates Monte Carlo experiment.

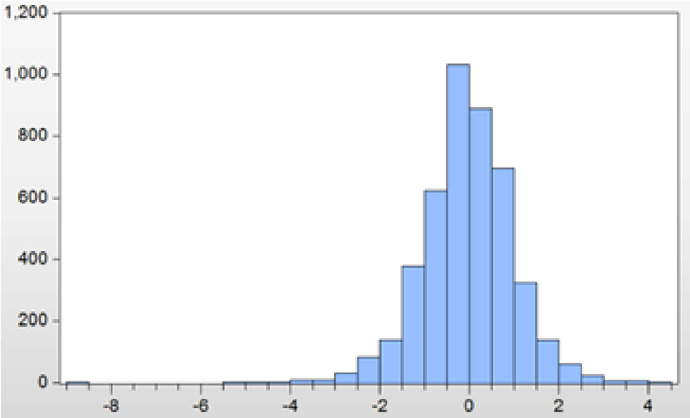

Figure no. 6 – Histogram of standardized residual series
Source: own processed based on Financeyahoo (2017)

5. CONCLUSION

The paper provides the two alternative approaches to stock price simulations known as bootstrap experiment and Monte Carlo experiment. These two methods are applied on the derivation of the value of European option and Asian option. As majority financial time series do not exhibit fixed variance, there are considered two distinct cases separately – with homoscedastic and with heteroscedastic error term. The bootstrap experiment using homoscedastic error term showed that the European option has higher value and higher variance than Asian option. The pitfall of this experiment is that does not take the assumption about distribution into consideration and in spite of that there are used data from fifteen years ago. This pitfall is compensated in the second simulation, bootstrap experiment with heteroscedastic error terms. In this experiment the data, which are not relevant are neglected. The values, which occurred very long time ago and exercised high volatility are hardly relevant for the stock prices in next twenty days. The results of the experiment are similar like in the previous case. The European option had higher value and higher volatility than Asian option. Third experiment is based on the different algorithm. In comparison with the previous models, Monte Carlo simulation using homoscedastic error term need to impose the assumption about the normal distribution. The output from the test showed that the data does not have normal distribution; therefore this experiment is not reliable. The results of the experiment is similar to the previous cases, thus the European option has higher value and also higher volatility than Asian option. The last experiment attempts to solve the problem with the distribution from the previous experiment. Monte Carlo simulation using heteroscedastic error term generates the stock prices for the next twenty days also based on the assumption about normal distribution, but firstly the volatility must be modelled. For the change the volatility from heteroscedastic to homoscedastic the econometric model GARCH (1,1) were used. Imposing GARCH, the series were converted to the homoscedastic, but the tests revealed that in spite of this conversion, the data does not have normal distribution. Although the pattern is also in this case the same, the conclusions are not relevant because of the wrong distributional assumption.

The initial hypothesis about the heteroscedastic pattern in the data (H1) was confirmed. The volatility in the data series indeed varies over the time. Based on the experimental results, it can be concluded that the initial hypothesis about the higher value of the European option than Asian option (H2) was confirmed. Because Asian option is calculated as the average of the prices during the life time of the option, its value tend to be on average smaller than the value at the day of expiration, which is used for the calculation of European option. Following the same mathematical logic, the value of the models with heteroscedastic error terms exhibit higher value than the models with homoscedastic error terms (H3). For example, imposing the blocked bootstrap value did not take valued with high volatility, which automatically decreased the average. The conclusion is that the all three null hypothesis were found to be correct (H1, H2 and H3).

Selection the best model was the next aim of the experiments. First of all, models where the assumption about the distribution must be imposed can be neglected. The assumption about the distribution is very strong and there is only the small chance that the investor knows the distribution. The tests of normality rejected the normal distribution of the data, which invalidates the both Monte Carlo simulations. Bootstrap simulations are therefore better, because the assumption about the distribution is not required. Bootstrap method with homoscedastic error term takes into calculation the data which are very old and their prediction power for the next twenty day is very poor. Currently the data does not exhibit high volatility therefore there is no reason to include the high volatility data into the simulation. Therefore the blocked bootstrap method is more reliable. The overall conclusion is that bootstrap experiment using heteroscedastic error term (blocked bootstrap) is the most reliable method for stock price simulation.

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